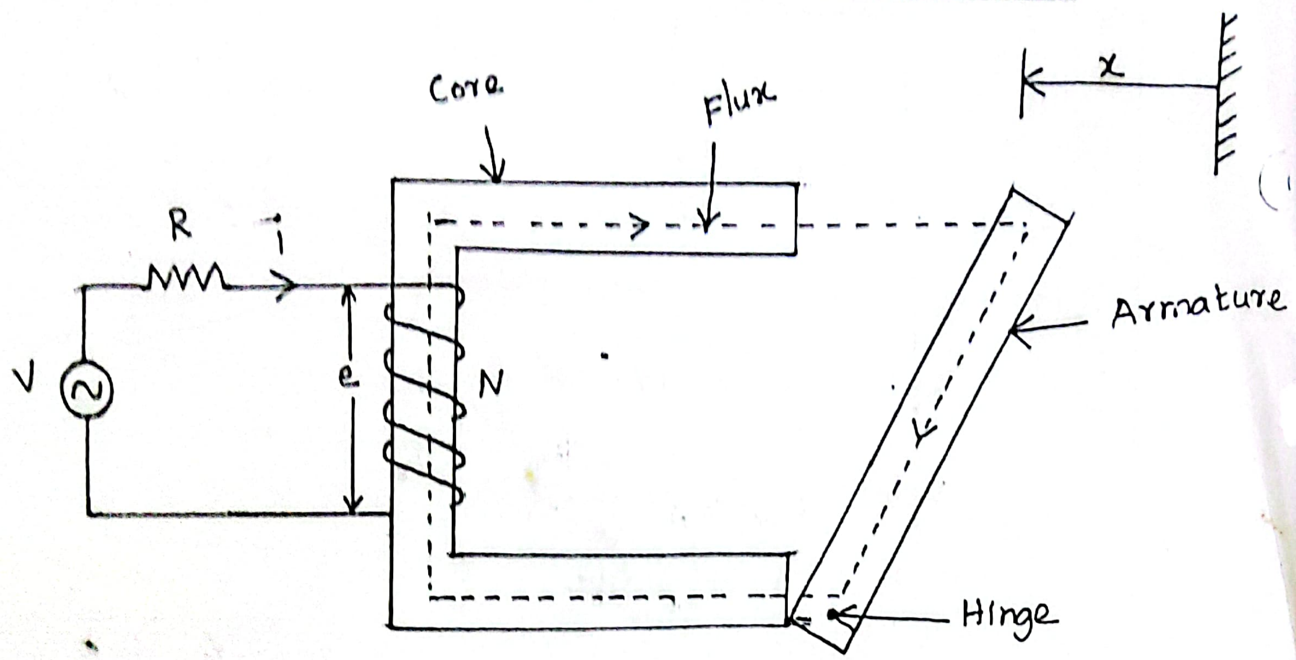
11. (a) (i) Derive the expression for the energy in singly excited magnetic field system. (7)

(ii) Write a technical note on doubly excited magnetic field system. (6)

Or

(b) (i) Develop the mathematical expression for the force and torque developed in the singly excited system. (7)

SINGLY EXCITED MAGNETIC SYSTEM



Consider, a simple electromagnetic relay as shown in Fig.

It has a movable portion which is hinged to one end called Armature

The non-moving part to which a loss less coil is wound having N turns, excited by a voltage V.

Assumption Made :

1. No loss of energy in the Magnetic core.
2. The whole of flux created by the core is confined to it. (No leakage flux)
3. Coil is lossless and Ideal. The resistance of the exciting coil is represented outside in a lumped fashion.

ELECTRIC ENERGY INPUT (W_e)

Flux linkage,
$$\lambda = N\phi \quad \text{———— (1)}$$

EMF,
$$e = \frac{d\lambda}{dt} \quad \text{———— (2)}$$

Applying KVL,

$$V = ir + e \qquad \text{———— (3)}$$

Sub (2) in (3)

$$V = ir + \frac{d\lambda}{dt}$$

X ing by i.dt on both sides

$$V.idt = ir.idt + \frac{d\lambda}{\cancel{dt}}.i\cancel{dt}$$

$$V.idt = i^2r.dt + id\lambda$$

$$V.idt - i^2r.dt = id\lambda$$

$$(V - ir)i\,dt = id\lambda$$

$$e\,idt = id\lambda$$

Thus electrical energy input in to ideal coil in a time dt :

$$\boxed{dW_e = id\lambda} \qquad \text{———— (4)}$$

W.K.T

$$\lambda = N\phi$$

$$\therefore (4) \Rightarrow \quad dW_e = id\,N\phi$$

$$dW_e = Ni\,d\phi$$

$$dW_e = MMF.d\phi$$

$$\boxed{dW_e = F\,d\phi} \qquad \text{———— (5)}$$

MAGNETIC FIELD ENERGY STORED (W_f)

- Consider the armature is held fixed.

If the armature is not allowed to move, The mechanical workdone is zero.

According to Energy Balance Equation,

$$dW_e = \cancel{dW_m}^{\,0} + dW_f$$

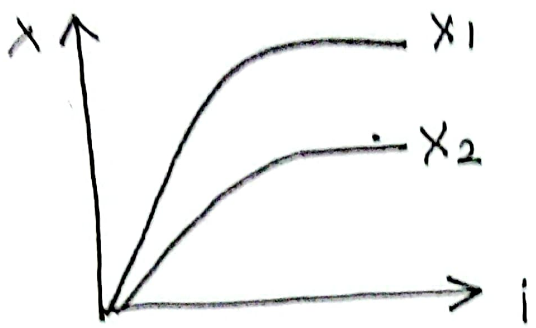
$$\therefore dW_e = dW_f$$

$dW_f = F \cdot d\phi \longrightarrow$ from (5)

$dW_f = i\,d\lambda \longrightarrow$ from (4).

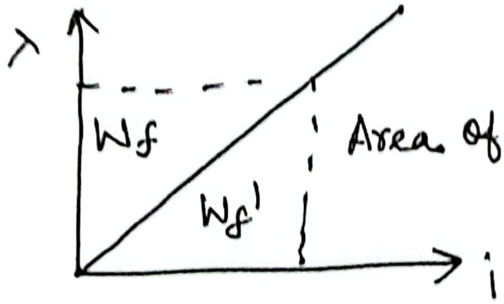
$$W_f = \int_0^{\lambda} i \cdot d\lambda$$

i-λ relation ship :



$x_1 > x_2$

iλ relation ship (linear case)



Area of rectangle = iλ

$W_f' \Rightarrow$ co. energy . It has no physical significance but it is important in obtaining magnetic forces.

$$\boxed{W_f = W_f' = \frac{1}{2}\lambda i}$$ ——— (15)

$$W_f = W_f' = \frac{1}{2} N\phi i$$ $[\lambda = N\phi]$

$$W_f = W_f' = \frac{1}{2} N i \phi$$

$$\boxed{W_f = W_f' = \frac{1}{2} F \phi}$$ ——— (16) $[F = NI]$

$$\boxed{W_f = W_f' = \frac{1}{2} S\phi^2}$$ ——— (17) $[F = \phi S]$

self Inductance,

$$L = \frac{\lambda}{i}$$

$$\Rightarrow i = \frac{\lambda}{L}$$ ——— (18)

substituting (18) in (15) we get

$$W_f = W_f' = \frac{1}{2} \lambda \cdot \frac{\lambda}{L}$$

$$\boxed{W_f = W_f' = \frac{1}{2} \cdot \frac{\lambda^2}{L}}$$ ——— (19)

$$W_f = W_f' = \frac{1}{2} \frac{\cancel{L^2} i^2}{\cancel{L}}$$ $[\lambda = Li]$

$$\boxed{W_f = W_f' = \frac{1}{2} L i^2}$$ ——— (20)

Expression for Coenergy Density.

$$\text{Coenergy Density} = \frac{W_f'}{\text{Volume}}$$

$$\omega_f' = \frac{W_f'}{\text{length} \times \text{Area}}$$

For linear case,

$$\omega_f = \omega_f'$$

(16) $\Rightarrow$

$$\omega_f = \omega_f' = \frac{\frac{1}{2} F \phi}{\text{length} \times \text{Area}}$$

$$= \frac{1}{2} \times \frac{F}{\text{length}} \times \frac{\phi}{\text{Area}}$$

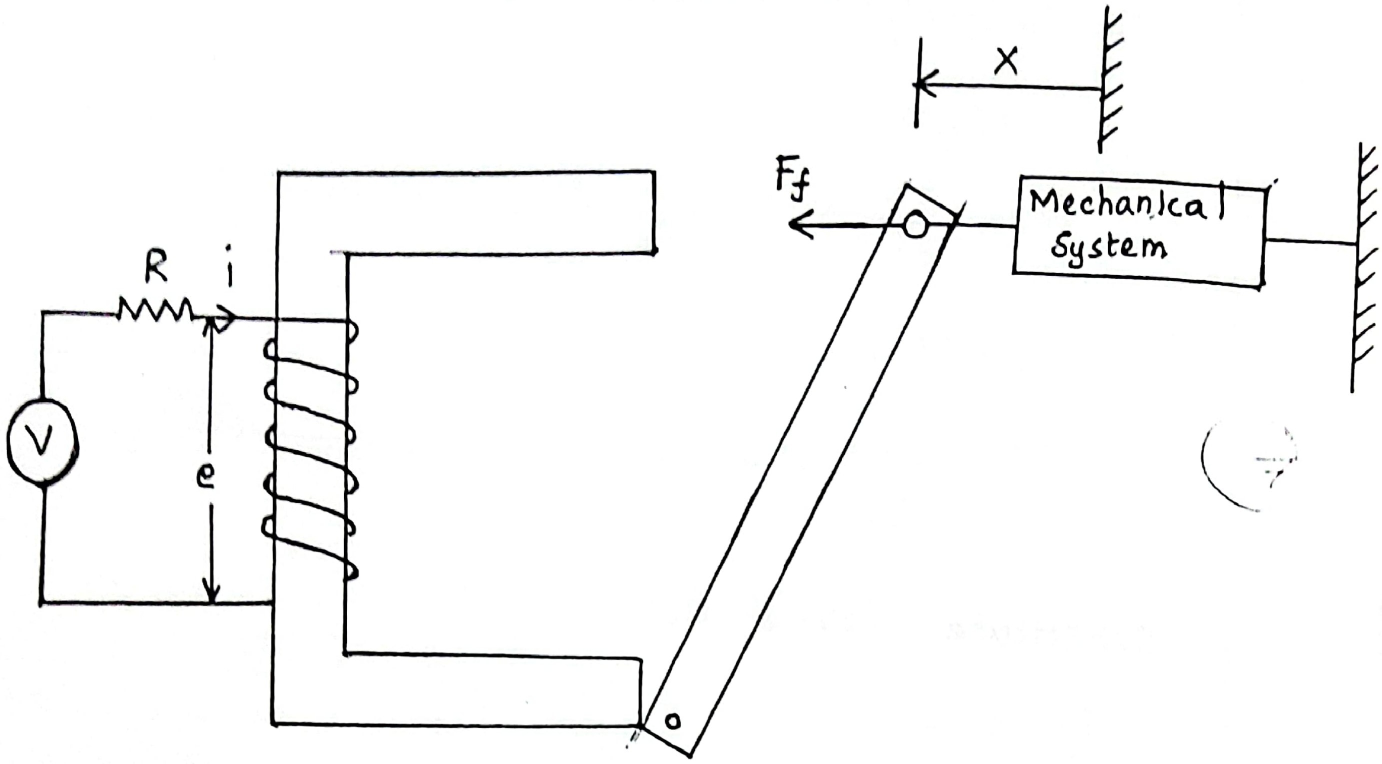
$$= \frac{1}{2} H B$$

$$\boxed{\omega_f = \omega_f' = \frac{1}{2} H B} \quad \text{———— (21)}$$

$$\omega_f = \omega_f' = \frac{1}{2} \mu H^2 \quad \text{———— (22)}$$

$$\omega_f = \omega_f' = \frac{1}{2} \frac{B^2}{\mu} \quad \text{———— (23)}$$

MECHANICAL FORCE (F_f)



When the coil is energized by the electric source, a magnetic flux is set up in it.

This magnetic field attracts the armature and hence produces a mechanical force F_f in the direction indicated.

This mechanical force F_f drives the mechanical system and causes a movement for a small distance dx.

The Mechanical Workdone,

$$\boxed{dW_m = F_f \cdot dx}$$

According to Energy Balance Equation,

$$dW_e = dW_m + dW_f$$

$$\Rightarrow dW_m = dW_e - dW_f$$

$$\Rightarrow F_f . dx = i d\lambda - dW_f$$

case i :
The Independent Variables are (i, x)

$\lambda(i, x)$

$$d\lambda = \frac{\partial \lambda}{\partial i} di + \frac{\partial \lambda}{\partial x} . dx$$

Also, $W_f(i, x)$

$$dW_f = \frac{\partial W_f}{\partial i} di + \frac{\partial W_f}{\partial x} . dx$$

$$F_f . dx = i\left[\frac{\partial \lambda}{\partial i} di + \frac{\partial \lambda}{\partial x} . dx\right] - \left[\frac{\partial W_f}{\partial i} . di + \frac{\partial W_f}{\partial x} . dx\right]$$

$$= i\frac{\partial x}{\partial i} di + i\frac{\partial \lambda}{\partial x} . dx - \left[\frac{\partial W_f}{\partial i} . di + \frac{\partial W_f}{\partial x} . dx\right]$$

$$F_f . dx = \left[\frac{i\,\partial x}{\partial i} - \frac{\partial W_f}{\partial i}\right] di + \left[\frac{i\,\partial \lambda}{\partial x} - \frac{\partial W_f}{\partial x}\right] dx$$

Equating the coefficients

$$\frac{i\,\partial \lambda}{\partial x} - \frac{\partial W_f}{\partial x} = F_f$$

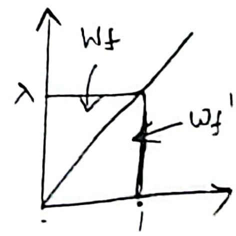
$$\Rightarrow \quad F_f = \frac{\partial}{\partial x}\left[i\lambda(i, x) - W_f(i, x)\right]$$

$$\boxed{F_f = \frac{\partial}{\partial x} W_f^{1}(i, x)} \quad \text{------ (29)}$$

$[W_f' = i\lambda - W_f]$

This expression for system in which i is independent variable.

It is a current excited system.



Case 2 :

The independent variables are (λ, x)

$$W_f = W_f(\lambda, x)$$

$$dW_f = \frac{\partial W_f}{\partial \lambda} . d\lambda + \frac{\partial W_f}{\partial x} . dx \quad \text{———— (30)}$$

substitute (30) in (26)

$$F_f . dx = i d\lambda - \frac{\partial W_f}{\partial \lambda} . d\lambda - \frac{\partial W_f}{\partial x} . dx$$

$$F_f . dx = \left[i - \frac{\partial W_f}{\partial \lambda} \right] d\lambda - \frac{\partial W_f}{\partial x} . dx$$

$$\boxed{F_f = - \frac{\partial W_f(\lambda, x)}{\partial x}} \quad \text{———— (31)}$$

This expression for System In which λ is independent Variable

It is a voltage controlled System.

Linear case :-

W.K.T

(29) $\Rightarrow$ $$F_f = \frac{\partial}{\partial x} W_f'(i, x)$$

$$F_f = \frac{\partial}{\partial x} \left[\frac{1}{2} L(x) i^2 \right] \quad \text{[From (20)]}$$

$$\boxed{F_f = \frac{1}{2} i^2 \frac{\partial L(x)}{\partial x}} \quad \text{———— (32)}$$

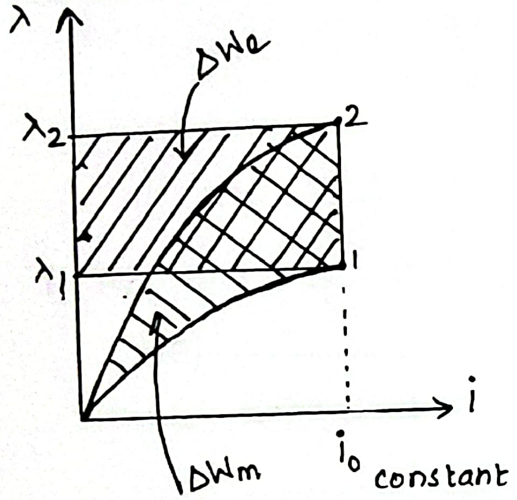
$$\boxed{F_f = \frac{1}{2} \left[\frac{\lambda}{L(x)} \right]^2 \frac{\partial L(x)}{\partial x}} \quad \text{———— (33)}$$

MECHANICAL ENERGY [Wm]

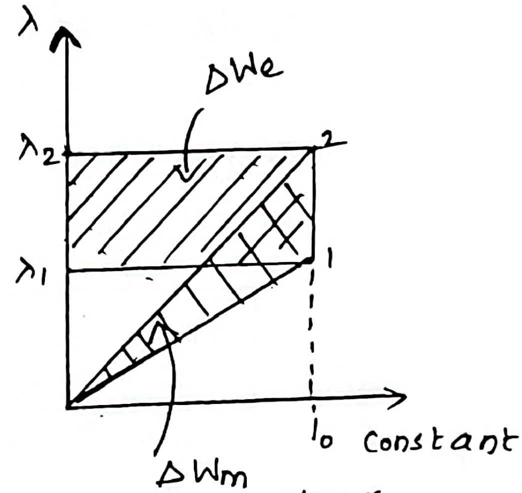
When the armature is allowed to move from x_1 to x_2, the mechanical energy output is

Case 1

coil current Constant



Non-linear



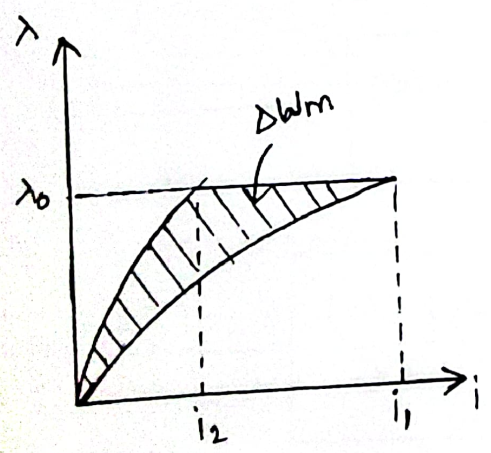
Linear

$$\boxed{\Delta W_m = \int_{x_1}^{x_2} F_f \cdot dx} \qquad (34)$$

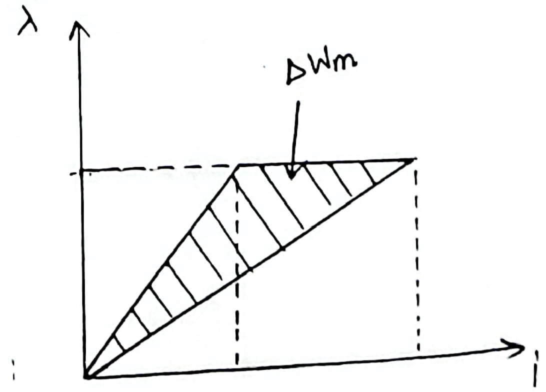
$$\boxed{\Delta W_m = \frac{1}{2} i_0 (\lambda_2 - \lambda_1)} \qquad (35)$$

Case 2

The flux linkages Constant



Non-linear



linear

$$\Delta W_m = \int_{X_1}^{X_2} F_f \cdot dx$$

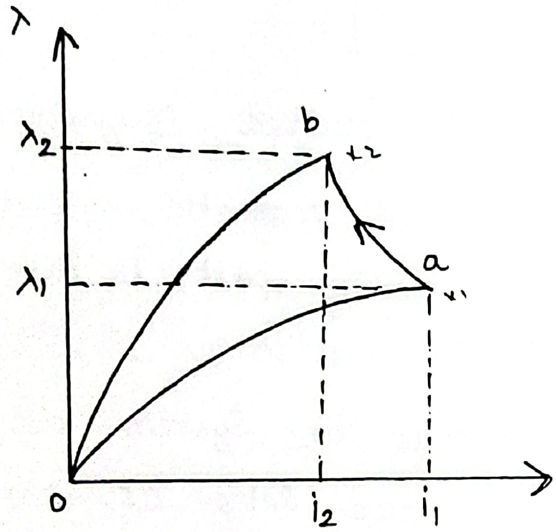
$$\boxed{\Delta W_m = \frac{1}{2} \lambda_0 (i_1 - i_2)} \quad \text{——— (36)}$$

Practical case

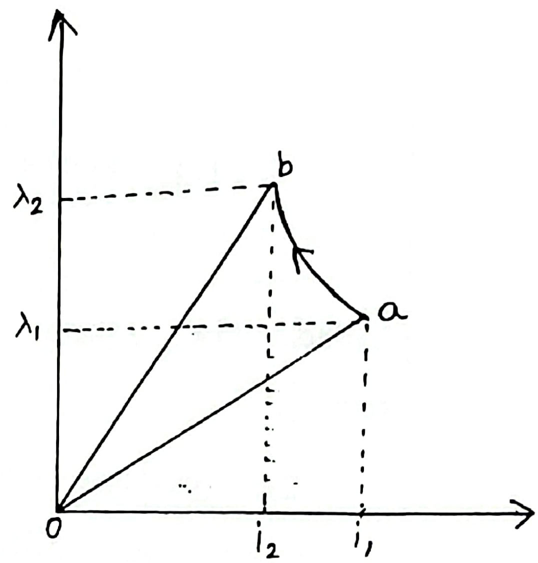
Armature moves from X_1 to X_2

current decreases from i_1 to i_2

Fluxlinkage increases from λ_1 to λ_2



Non linear



Liner

Mechanical Energy output,

$$\boxed{\Delta W_m = \text{Area}(oab)}$$

X——— X ——— X ——— X

MULTIPLY - EXCITED MAGNETIC SYSTEM

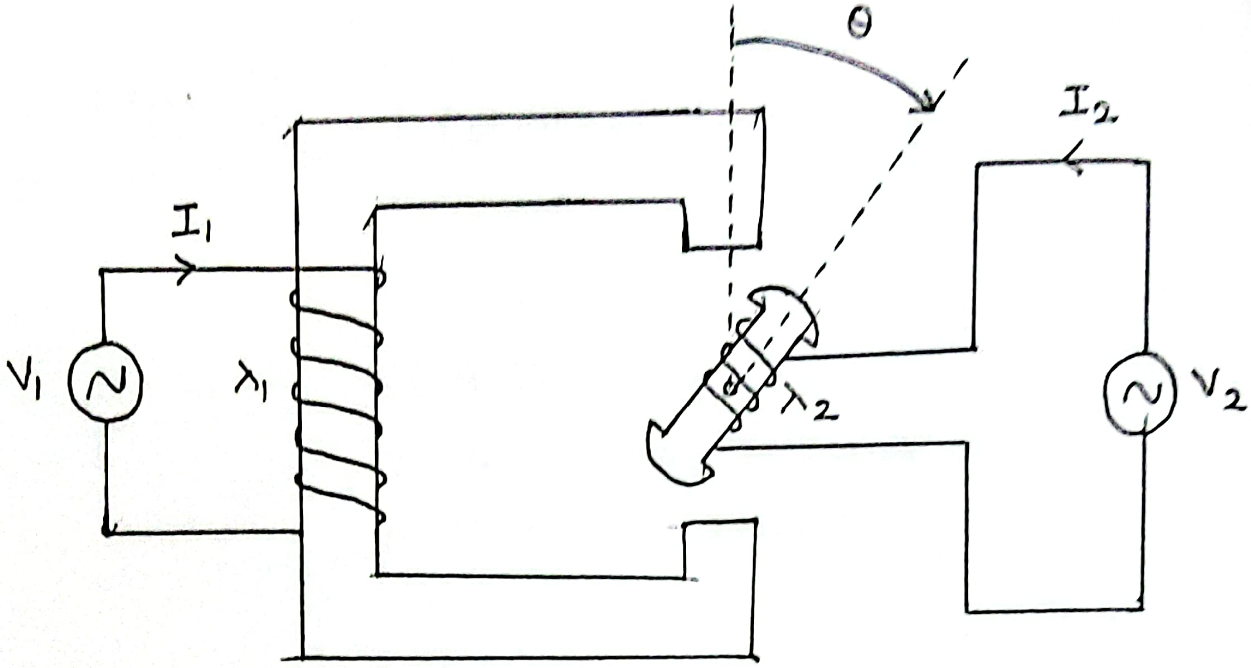


Fig shows the Doubly exited Magnetic System.

The fixed part of the magnetic system is called the stator and the moving part is called the rotor.

In Multiply exited system, the System can be described in either of the two sets of three Independent Variables $(\lambda_1, \lambda_2, \theta)$ or (i_1, i_2, θ)

Case1:

Independent Variables $(\lambda_1, \lambda_2, \theta)$

$$\boxed{T_f = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}} \quad \text{—— (37)} \qquad [\text{From } (31)]$$

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 \, d\lambda_1 + \int_0^{\lambda_2} i_2 \, d\lambda_2 \quad \text{—— (A)} \qquad [\text{From } (8)]$$

let

$L_{11} \rightarrow$ Self Inductance of stator

$L_{22} \rightarrow$ Self Inductance of rotor

$L_{12} = L_{21} \rightarrow$ Mutual Inductance between stator and Rotor.

For the Doubly excited circuit shown, the flux linkages can be represented as

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \quad \text{------------} \quad (38)$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \quad \text{------------} \quad (39)$$

Solve eqn (38) & (39) & find i_1 & i_2

$$(38) \times L_{12} \Rightarrow \lambda_1 L_{12} = \cancel{L_{11} L_{12} i_1} + L_{12}^2 i_2$$

$$(39) \times L_{11} \Rightarrow \overset{(-)}{\lambda_2 L_{11}} = \overset{(-)}{\cancel{L_{11} L_{21} i_1}} + \overset{(-)}{L_{11} L_{22} i_2}$$

$$\lambda_1 L_{12} - \lambda_2 L_{11} = L_{12}^2 i_2 - L_{11} L_{22} i_2$$

$$\lambda_1 L_{12} - \lambda_2 L_{11} = \left(L_{12}^2 - L_{11} L_{22}\right) i_2$$

$$\Rightarrow i_2 = \frac{\lambda_1 L_{12} - \lambda_2 L_{11}}{L_{12}^2 - L_{11} L_{22}}$$

$$\Rightarrow i_2 = \left[\frac{L_{12}}{L_{12}^2 - L_{11} L_{22}}\right] \lambda_1 - \left[\frac{L_{11}}{L_{12}^2 - L_{11} L_{22}}\right] \lambda_2$$

$$\Rightarrow i_2 = \left[\frac{-L_{12}}{L_{11} L_{22} - L_{12}^2}\right] \lambda_1 + \left[\frac{L_{11}}{L_{11} L_{22} - L_{12}^2}\right] \lambda_2$$

------------ (40)

$$(38) \times L_{22} \Rightarrow \lambda_1 L_{22} = L_{11} L_{22} i_1 + \cancel{L_{12} L_{22} i_2}$$

$$(39) \times L_{12} \Rightarrow \overset{(-)}{\lambda_2 L_{12}} = \overset{(-)}{L_{12} L_{21} i_1} \overset{(-)}{+} \cancel{L_{12} L_{22} i_2}$$

$$\lambda_1 L_{22} - \lambda_2 L_{12} = L_{11} L_{22} i_1 - L_{12}^2 i_1$$

$$\Rightarrow \lambda_1 L_{22} - \lambda_2 L_{12} = \left[L_{11} L_{22} - L_{12}^2\right] i_1$$

$$\Rightarrow i_1 = \frac{\lambda_1 L_{22} - \lambda_2 L_{12}}{L_{11} L_{22} - L_{12}^2}$$

$$\Rightarrow i_1 = \left[\frac{L_{22}}{L_{11} L_{22} - L_{12}^2}\right] \lambda_1 - \left[\frac{L_{12}}{L_{11} L_{22} - L_{12}^2}\right] \lambda_2 \quad \text{——— (41)}$$

generally,

$$i_1 = \beta_{11} \lambda_1 + \beta_{12} \lambda_2 \quad \text{——— (42)}$$

$$i_2 = \beta_{12} \lambda_1 + \beta_{22} \lambda_2 \quad \text{——— (43)}$$

comparing (40), (41), (42) & (43) we get

$$\beta_{11} = \frac{L_{22}}{L_{11} L_{22} - L_{12}^2}$$

$$\beta_{12} = \beta_{21} = \frac{-L_{12}}{L_{11} L_{22} - L_{12}^2}$$

$$\beta_{22} = \frac{L_{11}}{L_{11} L_{22} - L_{12}^2}$$

$$W_f(\lambda_1,\lambda_2,\theta) = \int_0^{\lambda_1} [\beta_{11}\lambda_1 + \beta_{12}\lambda_2]\, d\lambda_1 + \int_0^{\lambda_2} [\beta_{12}\lambda_1 + \beta_{22}\lambda_2]\, d\lambda_2$$

$$= \left[\frac{\beta_{11}\lambda_1^2}{2}\right]_0^{\lambda_1} + \int_0^{\lambda_1} \beta_{12}\lambda_2\, d\lambda_1 + \int_0^{\lambda_2} \beta_{12}\lambda_1\, d\lambda_2 + \left[\frac{\beta_{22}\lambda_2^2}{2}\right]_0^{\lambda_2}$$

$$= \beta_{11}\frac{\lambda_1^2}{2} + \beta_{22}\frac{\lambda_2^2}{2} + \beta_{12}\lambda_1\lambda_2$$

$$\boxed{W_f(\lambda_1,\lambda_2,\theta) = \frac{1}{2}\beta_{11}\lambda_1^2 + \frac{1}{2}\beta_{22}\lambda_2^2 + \beta_{12}\lambda_1\lambda_2} \quad \text{——— (44)}$$

<u>case 2 :-</u>

<u>Independent Variables (i_1, i_2, θ)</u>

$$\boxed{T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}} \quad \text{——— (45)}$$

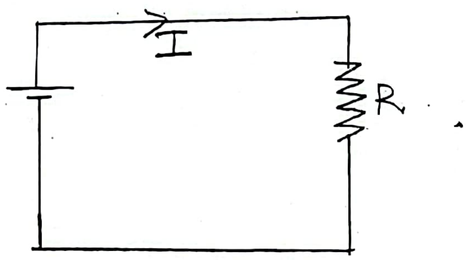
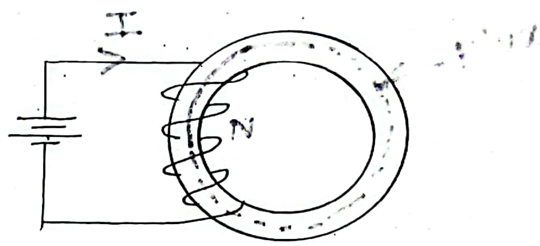
$$W_f'(i_1,i_2,\theta) = \int_0^{i_1} \lambda_1\, di_1 + \int_0^{i_2} \lambda_2\, di_2 \quad \text{——— (46)}$$

$$= \int_0^{i_1} [L_{11}i_1 + L_{12}i_2]\, di_1 + \int_0^{i_2} [L_{12}i_1 + L_{22}i_2]\, di_2$$

$$\boxed{W_f'(i_1,i_2,\theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1 i_2} \quad \text{——— (47)}$$

ANALOGY BETWEEN ELECTRIC AND MAGNETIC CIRCUITS (SIMILARITIES)

Refer also Pg.NO.1.13 Gnanavadivel.

ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
Path traced by the current is called Electric circuit 	Path traced by the Magnetic flux is called as Magnetic circuit 
E.M.F Is the driving force in electric circuit, the unit is volts. (V)	M.M.F is the driving force in the magnetic circuit, the unit is. Ampere Turns (AT).
current (I)	Flux (ϕ)
Resistance (R) oppose the flow of current. unit is ohm	Reluctance (S) oppose the flux in the magnetic path. unit is Aturns/weber.
$R = \frac{\rho l}{A}$	$S = \frac{l}{\mu A}$
current $I = \frac{emf}{Resistance}$	flux $\phi = \frac{m.m.F}{Reluctance}$.
The current density $\delta = \frac{I}{A}$ A/m²	The flux density $B = \frac{\phi}{A}$ Wb/m²
conductance $= \frac{1}{R}$	permeance $= \frac{1}{S}$.

current → flux
Emf → mmf
Resistance → Reluctance,

as a motor.

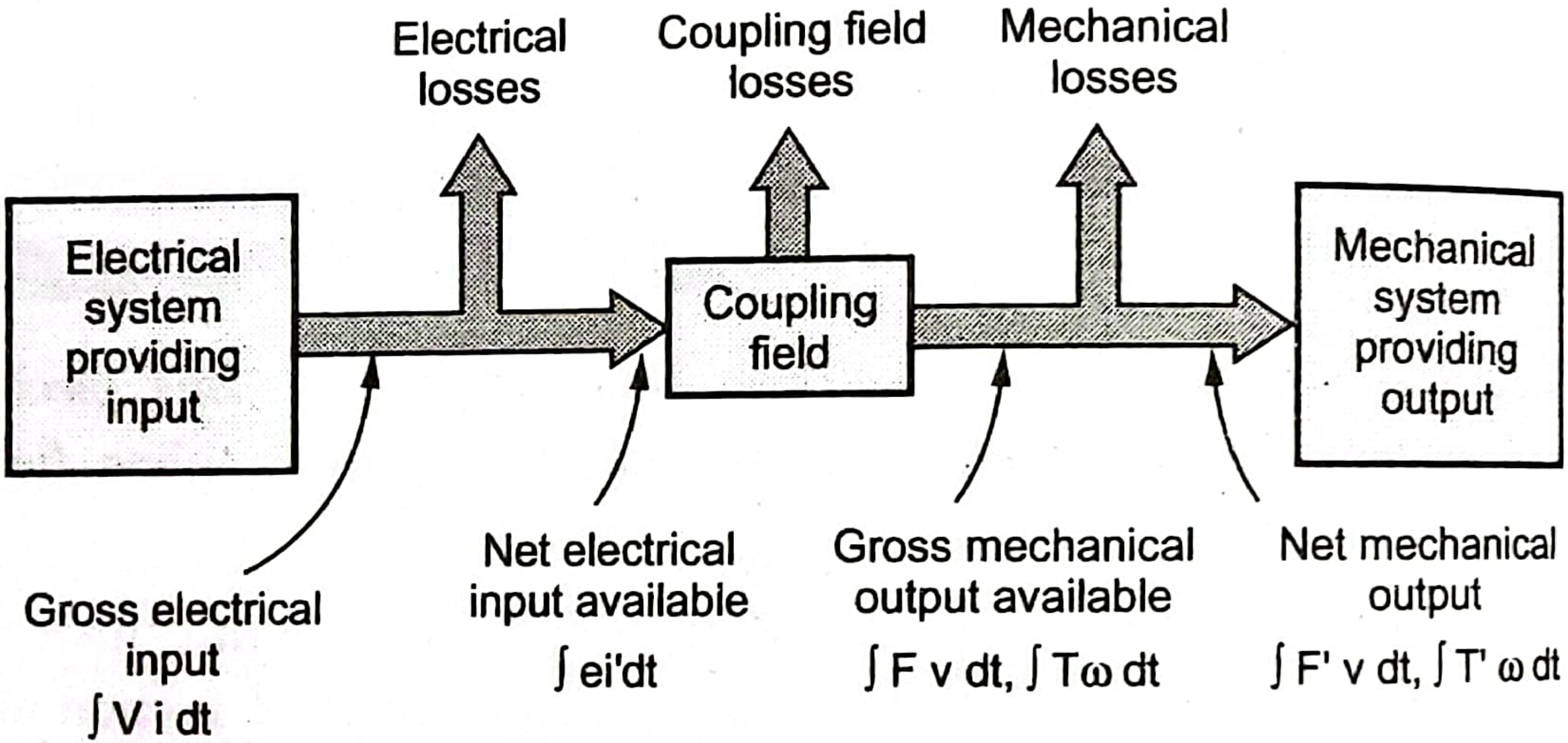


Fig. 5.4.2 Energy flow in electromechanical energy conversion device as a motor

FIRST LAW :-

Whenever the magnetic flux linked with a circuit changes, an e.m.f is Induced in it.

(or)

Whenever a conductor cuts magnetic flux, an emf is induced in that Conductor.

SECOND LAW :-

The magnitude of induced emf is equal to the rate of change of flux - linkages.

Mathematically, $\boxed{e = N\frac{d\phi}{dt}}$

LENZ'S LAW :-

Lenz's law states that " The direction of the induced emf due to electromagnetic induction is such that the current set up by it tends to oppose the change which causes the induced emf .

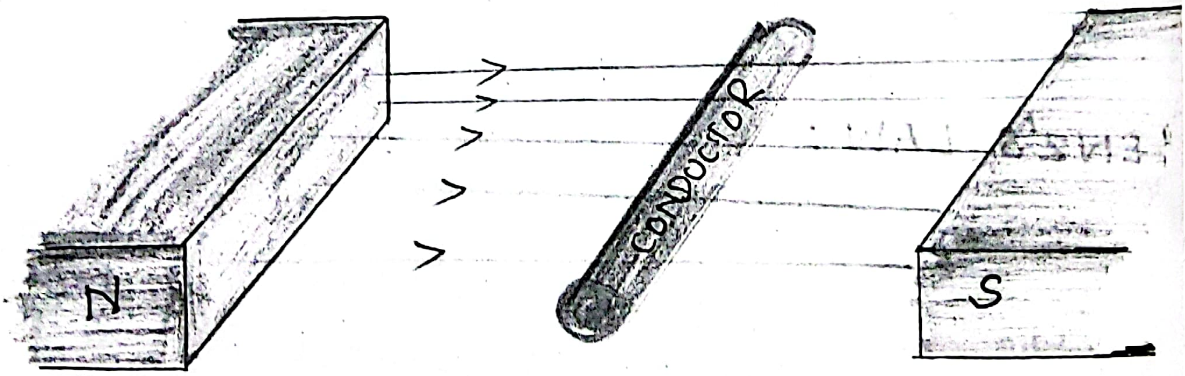
INDUCED E.M.F

Depending up on the nature of methods, the induced emf is classified,

1) Dynamically induced emf
2) Statically induced emf.

DYNAMICALLY INDUCED EMF

An induced e.m.f which is due to physical movement of conductor with respect to flux or movement of magnet with respect to stationary conductor is called dynamically induced emf.

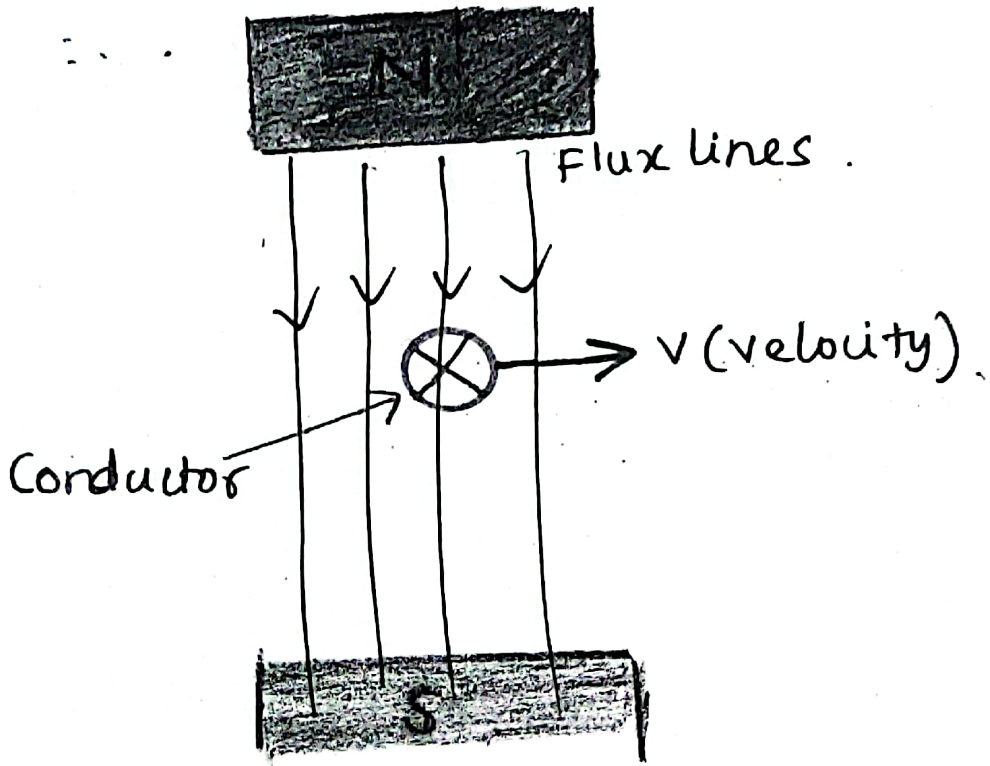


case(i) Movement of the conductor

case(ii) Movement of the Conductor

Consider a stationary magnetic field of flux density of B Wb/m^2 with direction shown in Fig below

A circular conductor is placed in this field. let 'l' be the length of the conductor in metres.



Consider a conductor moving with velocity v m/s. such that its plane of motion of direction of velocity is perpendicular to the direction of flux as shown in Fig.

In a time 'dt' seconds, the distance moved is 'dx' metres.

Area swept by the conductor = $l.dx$

$\therefore$ Flux cut by the conductor = Flux density × Area swept

$$\boxed{d\phi = B \times l\,dx} \quad \text{——(1)}$$

According to Faraday's law of electromagnetic induction, e.m.f induced in the conductor is given by

$$e = N.\frac{d\phi}{dt}$$

$$e = \frac{d\phi}{dt} \quad \text{——(2)}$$ [consider conductor has 1 turn]

sub. (1) in (2) we get

$$e = \frac{B \times l\,dx}{dt}$$

$$e = Bl\,\frac{dx}{dt}$$

$\frac{dx}{dt}$ = Rate of change of displacement.

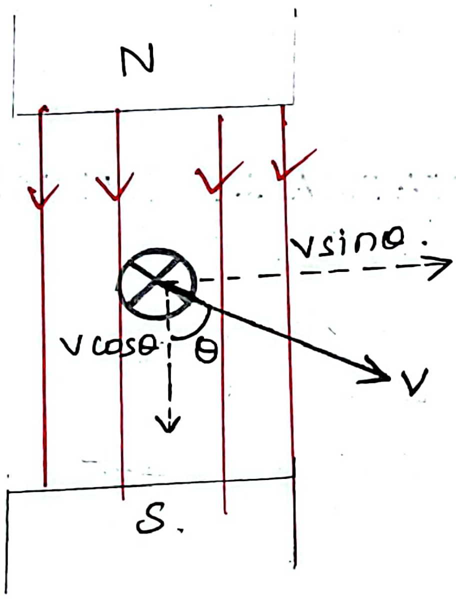
= linear Velocity of the conductor.

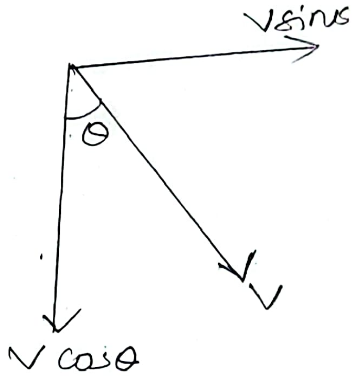
= V

$$\boxed{e = Blv} \quad \text{volts.}$$

This is induced emf when plane of motion is right angles to the plane of flux.

If the conductor is moving with a velocity v at an certain angle θ measured with respect to the direction of the field. as shown in Fig.





Here the Component of velocity which is v sinθ is perpendicular to the direction of flux and hence responsible for the induced e.m.f.

The other Component $V\cos\theta$ ~~Vsinθ~~ is parallel to the plane of flux and hence will not contribute to the dynamically induced emf.

under this condition magnitude of induced emf is given by

$$\boxed{e = Blv \sin\theta} \text{ volts.}$$

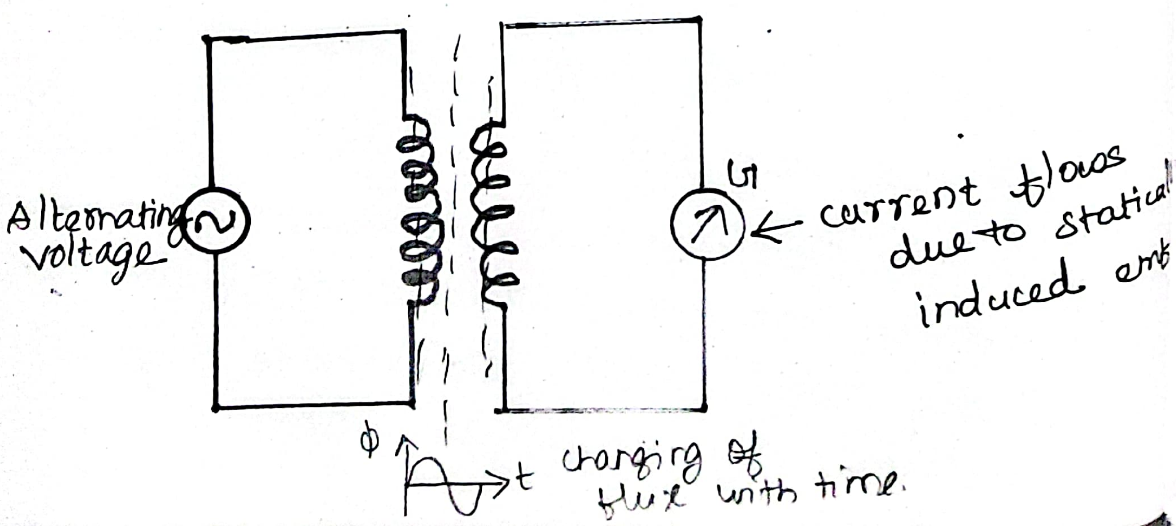
The direction of Induced emf is determined by fleming Right hand rule

If fore finger → direction of flux lin
Thumb → direction of motion

Then
middle finger → direction of Induced e.m.f

STATICALLY INDUCED E.M.F

When change in flux lines can be achieved without physically moving the conductor or the magnet. (or) Induced emf in a conductor which is without physical movement of coil or a magnet is called statically induced e.m.f



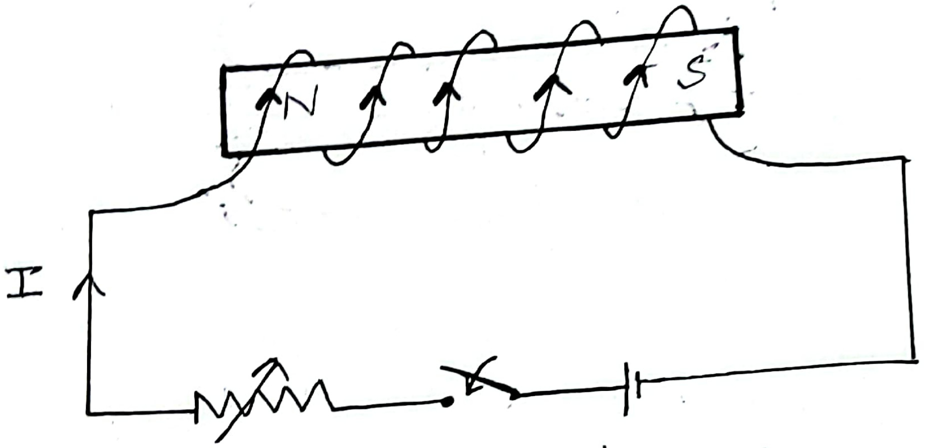
The statically Induced e.m.f is further classified as

1) self Induced e.mf

2) Mutually induced e.m.f

SELF INDUCED EMF :-

"Self Induced emf is the e.m.f induced in a coil due to the change of its own flux linked"



When the coil carries a current, a flux will be setup in it.

Now If the current 'I' is changed with the help of variable resistance, then flux produced will also change.

Hence according to faraday's law, due to the rate of change of flux linkage there is an induced emf in the coil.

(ie. Emf induced in the coil due to the change of its own flux linkage with it)

SELF INDUCTANCE :

According to Lenz's law the direction of this induced emf will be so as to oppose the cause producing it.

The property of the coil which opposes any change in the current passing through is called self Inductance.

EXPRESSIONS FOR SELF INDUCTANCE (L)

The self Induced emf 'e' in the coil is given by

$$\boxed{e = N \cdot \frac{d\phi}{dt}} \quad \text{—— (1)}$$

$$e = \frac{d(N\phi)}{dt}$$

Flux linkage $\lambda = N\phi$

Flux linkage $N\phi$ depends on current (proportional)

$$\Rightarrow \therefore e \propto \frac{dI}{dt}$$

$$\Rightarrow \boxed{e = L \cdot \frac{dI}{dt}} \quad \text{—— (2)}$$

where 'L' is a constant called self-Inductance of the coil. The unit of Inductance is henry (H).

A coil has an inductance of 1 H if an e.m.f of 1 volt is induced in it when current through it changes at the rate of 1 ampere per second.

Now

$$(1) \Rightarrow e = N.\frac{d\phi}{dt} = \frac{d(N\phi)}{dt}$$

$$(2) \Rightarrow e = L.\frac{dI}{dt} = \frac{d(LI)}{dt}$$

$$\Rightarrow \quad N\phi = LI$$

$$\boxed{L = \frac{N\phi}{I}} \quad \text{———} (3)$$

$$\boxed{L = \frac{N\phi}{I}}$$ ——— (3)

W.K.T

$$\phi = \frac{m.m.f}{Reluctance}$$

$$\boxed{\phi = \frac{NI}{S}} \quad \text{——— (4)}$$

Substituting (4) in (3)

$$L = \frac{N \cdot N\cancel{I}}{S \times \cancel{I}}$$

$$\boxed{L = \frac{N^2}{S}} \quad \text{——— (5)}$$

W.K.T

$$S = \frac{l}{\mu A} \quad \text{——— (6)}$$

Sub (6) in (5).

$$L = \frac{N^2}{\left(\frac{l}{\mu A}\right)}$$

$$\boxed{L = \frac{N^2 \mu A}{l}} \quad \text{——— (6)}$$

$$\boxed{L = \frac{N^2 \mu_0 \mu_r A}{l}} \quad \text{Henries.}$$

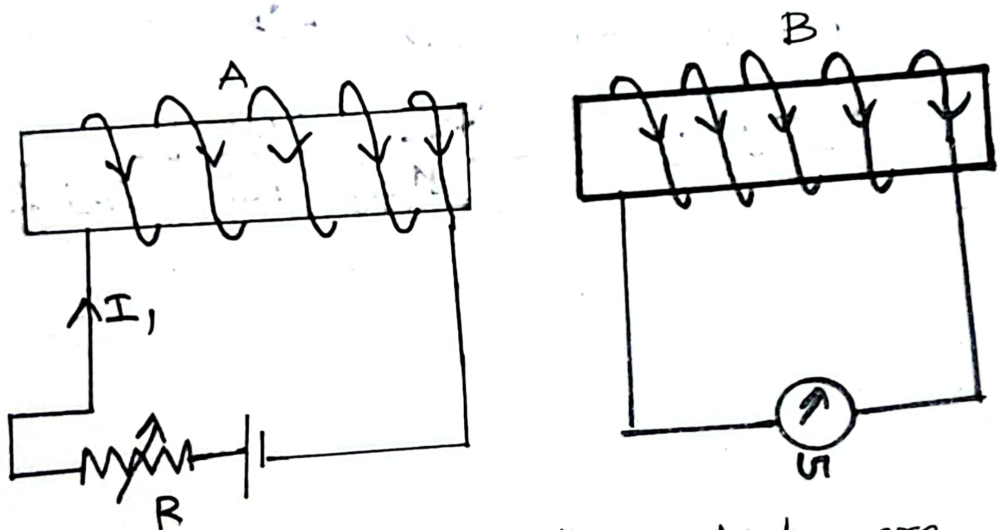
where,

$L \rightarrow$ self Inductance

$A \rightarrow$ Area of cross section of the magnetic circuit.

$l \rightarrow$ length of the magnetic circuit

The EMF induced in a circuit due to the changing current in the neighbouring circuit is called mutually induced emf.



Consider two coils which are placed adjacent to each other as shown in Fig.

✓ When current through A flows produces flux ϕ. Part of this flux link with coil B as shown in Fig. This is called mutual flux.

✓ Now If current through coil A is changed by means of variable resistance Then flux ϕ changes. Due to this flux associated with coil B also changes.

As a result an emf is induced in coil B will be detected by Galvanometer G.

in the coil B

Here the mutually induced emf in the coil B is directly proportional to rate of change of current in first coil. A.

$$e \propto \frac{dI_1}{dt}$$

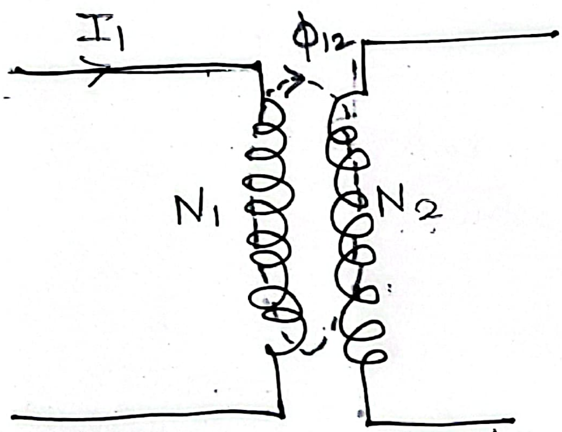
$$\boxed{e = M\frac{dI_1}{dt}}$$

where M is a Constant called mutual inductance between two coils.

The unit of Mutual inductance is henry (H).

Mutual inductance between two coils is 1 henry if current changing at the rate of 1 A/sec in one coil induces an e.m.f of 1V in the other coil.

<u>Expressions of the Mutual Inductance</u> (M)



ϕ_{12} is the part of the flux ϕ_1 Produced due to I_1

$$e_m = M \cdot \frac{dI_1}{dt}$$

$$\boxed{e_m = \frac{d(MI_1)}{dt}} \quad \text{——— (1)}$$

Also,

$$e_m = N_2 \cdot \frac{d\phi_{12}}{dt}$$

$$\boxed{e_m = \frac{d(N_2\phi_{12})}{dt}} \quad \text{——— (2)}$$

From (1) and (2).

$$MI_1 = N_2 \phi_{12}$$

$$\boxed{M = \frac{N_2 \phi_{12}}{I_1}} \quad \text{——— (A)}$$

ϕ_{12} is the part of the flux ϕ_1 produced due to I_1.

let 'K' be the fraction of ϕ_1 which is linking coil B.

ie) $K = \frac{\phi_{12}}{\phi_1}$ (called co-efficient of coupling or magnetic coupling).

coefficient of coupling is defined as the fraction of the total flux produced by one coil linking the other coil.

A⇒

$$\boxed{M = \frac{K N_2 \phi_1}{I_1}} \quad \text{——— (B)}$$

Mutual flux

$$\phi_1 = \frac{mmf}{Reluctance}$$

$$\phi_1 = \frac{N_1 I_1}{S} \quad \text{——— (C).}$$

sub (C) in (B).

$$M = \frac{K N_2 N_1 \cancel{I_1}}{\cancel{I_1} S}$$

$$\boxed{M = \frac{K N_1 N_2}{S}}$$

A⇒

$$\boxed{M = \frac{N_2 \phi_{12}}{I_1}} \quad \text{——— (D)}$$

This is when first coil carries current I_1 ~~producing flux ϕ_{12}~~ ~~A part of flux ϕ links with coil B.~~
Then ~~and~~ flux produced by coil A links w coil B.

* when second coil B carries current I_2 then flux produced by coil B links with coil A

$$\boxed{M = \frac{N_1 \phi_{21}}{I_2}} \quad \text{——— (E)}$$

From (I) & (II),

$$M \times M = \frac{N_2 \phi_{12}}{I_1} \times \frac{N_1 \phi_{21}}{I_2}$$

$$M^2 = \frac{N_1 N_2 \phi_{12} \phi_{21}}{I_1 I_2}$$

$$\boxed{K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}}$$

$$\Rightarrow \phi_{12} = K\phi_1$$
$$\phi_{21} = K\phi_2$$

Now,

$$M^2 = \frac{N_1 N_2 K\phi_1 \cdot K\phi_2}{I_1 I_2}$$

$$M^2 = \frac{K^2 N_1 N_2 \phi_1 \phi_2}{I_1 I_2} \qquad \left[L = \frac{N\phi}{I}\right]$$

$$M^2 = K^2 L_1 L_2$$

$$M = K\sqrt{L_1 L_2}$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

K is always positive & its maximum value is 1

when K = 1

$$\boxed{M = \sqrt{L_1 L_2}}$$

Series Magnetic circuit

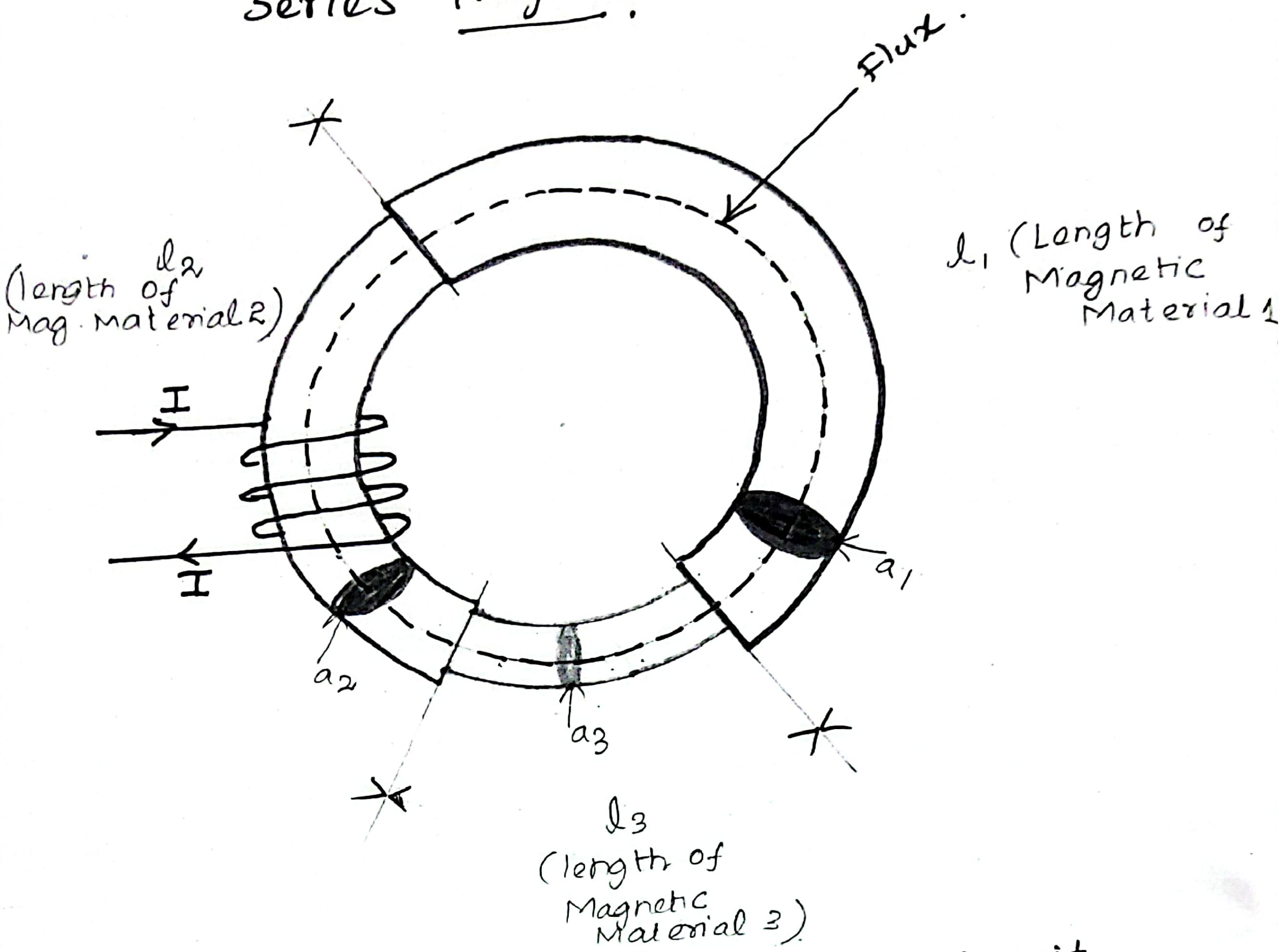


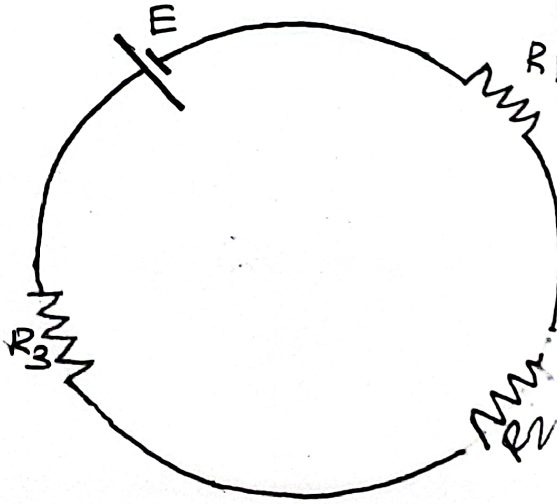
Fig. A series Magnetic circuit.

Consider a circular ring made up of different materials of length l_1, l_2, l_3 and with cross-sectional areas a_1, a_2 and a_3 and with obsolute permeabilities μ_1, μ_2, μ_3 as shown in Fig.

The coil wound on the ring has N turns carries a current of I amperes.

$\therefore$ MMF = IN

The analogous electric circuit can be drawn as.



The total resistance of the above electrical circuit is $R_1 + R_2 + R_3$.

||ly

1) Total Reluctance, $S_T = S_1 + S_2 + S_3$

$$= \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

3) Total flux, $\phi_{Total} = \frac{\text{Total m.m.f}}{\text{Total reluctance}}$

$$= \frac{NI}{S_T}$$

2) M.M.F Total = m.mf 1 + m.m.f 2 + m.mf 3

$$NI = S_1\phi + S_2\phi + S_3\phi$$
$$= H_1 l_1 + H_2 l_2 + H_3 l_3$$

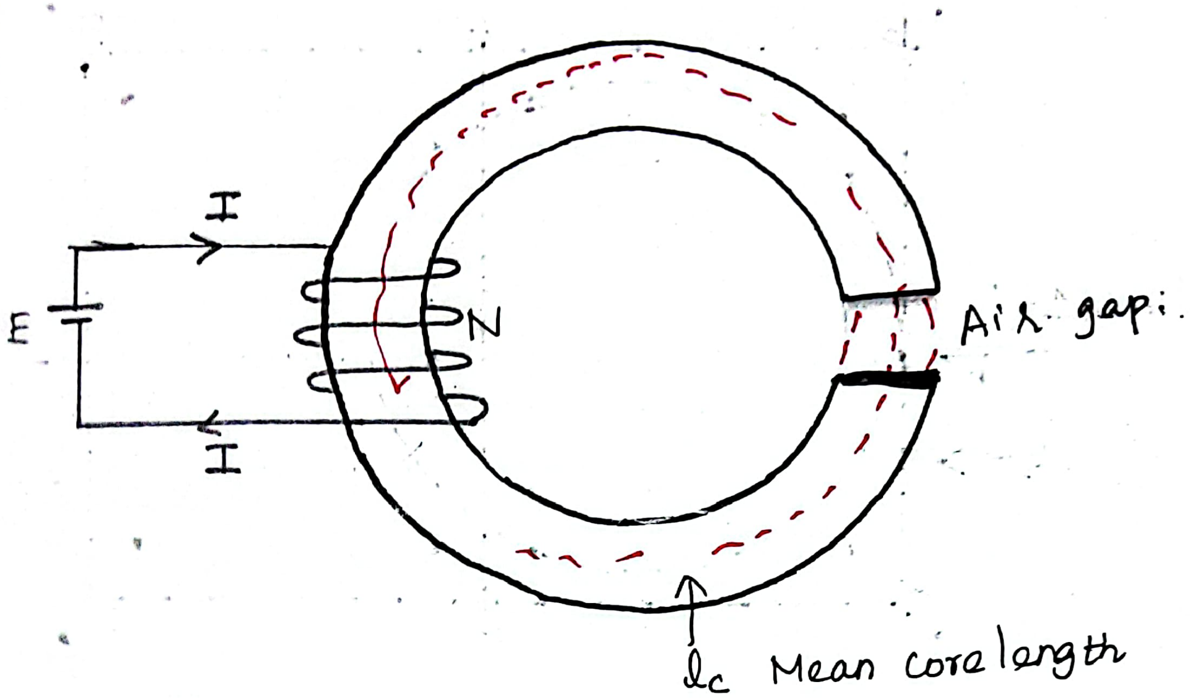
$$H_1 = \frac{B_1}{\mu_1}, \quad H_2 = \frac{B_2}{\mu_2}, \quad H_3 = \frac{B_3}{\mu_3}.$$

✓ The magnetic flux through all parts is same.

✓ The equivalent reluctance is the sum of different parts.

✓ The resultant m.mf necessary is sum of the m.mf in each individual part.

Series circuit with Air gap :



Total m.m.f $= NI$ AT

Reluctance S_i of iron path $= \dfrac{l_c}{\mu A_c}$

Reluctance of airgap $S_g = \dfrac{l_g}{\mu_0 A_c}$

Total reluctance $S_T = S_i + S_g$

$$\phi = \frac{m.m.f}{Reluctance} = \frac{NI}{S_T}$$

$$\boxed{NI = S_i \phi + S_g \phi}$$

Example 1.18.5 *A ring composed of three sections. The cross section area is 0.001 m^2 for each section. The mean arc length are l_a = 0.3 m, l_b = 0.2 m, l_c = 0.1 m, an air gap length of 0.1 mm is cut in the ring, μ_r for sections a, b and c are 5000, 1000 and 10000 respectively. Flux in the air gap is 7.5×10^{-4} Wb. Find i) m.m.f. ii) Exciting current if the coil has 100 turns iii) Reluctance of the sections.* **AU : Dec.-11, Marks 16**

Solution : a= 0.001 m^2 same, for all sections,

$$\phi = 7.5\times10^{-4} \text{ Wb},$$

$$N = 100$$

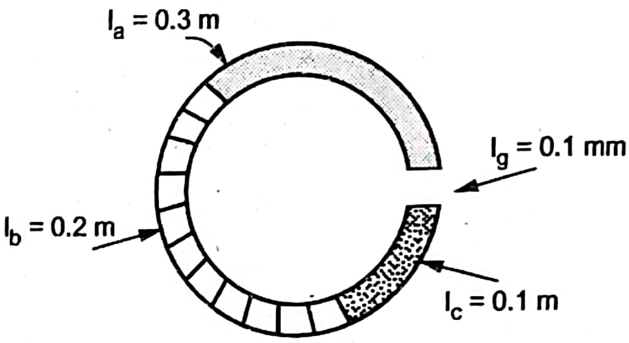


Fig. 1.18.8

Reluctances for the sections,

$$S_a = \frac{l_a}{\mu_0 \mu_r a} = \frac{0.3}{4\pi\times10^{-7}\times5000\times0.001}$$

$$= \mathbf{47746.4829\ AT/Wb}$$

$$S_b = \frac{l_b}{\mu_0 \mu_r a} = \frac{0.2}{4\pi\times10^{-7}\times1000\times0.001} = \mathbf{159154.9431\ AT/Wb}$$

$$S_c = \frac{l_c}{\mu_0 \mu_r a} = \frac{0.1}{4\pi\times10^{-7}\times10000\times0.001} = \mathbf{7957.7471\ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{0.1\times10^{-3}}{4\pi\times10^{-7}\times0.001} = \mathbf{79577.4715\ AT/Wb}$$

$\therefore$ Total reluctance = $S_T = S_a + S_b + S_c + S_g$ = **294.4366 × 10^3 AT/Wb**

$$\phi = \frac{\text{m.m.f.}}{S_T} \quad \text{i.e.} \quad 7.5\times10^{-4} = \frac{\text{m.m.f.}}{294.4366\times10^3}$$

$\therefore$ m.m.f. = **220.8275 AT**

$$\text{m.m.f.} = NI \quad \text{i.e.} \quad I = \frac{220.8275}{100} = \mathbf{2.2082\ A}$$

Example 1.18.4 *For the magnetic circuit shown in Fig. 1.18.7 determine the current required to establish a flux density of 0.5 T in the air gap.*

AU : Dec.-10, Marks 12

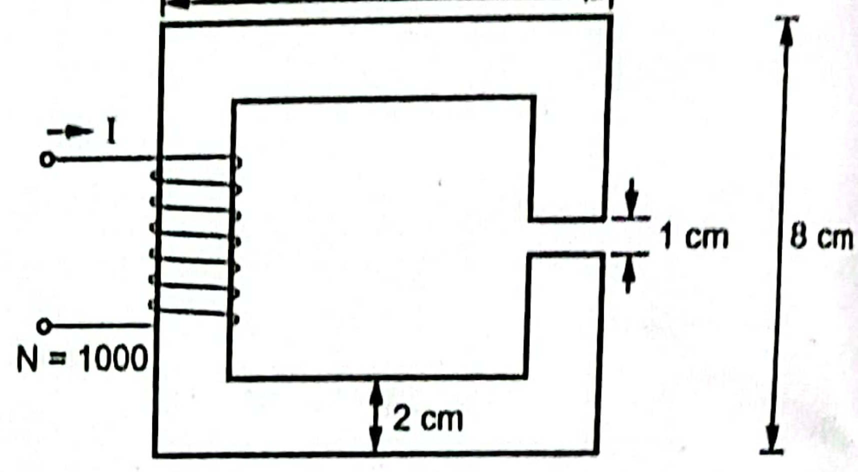


Fig. 1.18.7

Solution : N = 1000, B = 0.5 T, a = 2×2 = 4 cm^2, l_g = 1 cm

$$l_i = \text{Length of iron path} = 8 + 8 + 6 + 5 = 27 \text{ cm}$$

$$\mu_{core} = 5000\,\mu_0 \quad \text{i.e.} \quad \mu_r = 5000$$

$$\phi = B \times a = 0.5 \times 4 \times 10^{-4} = 0.2 \text{ mWb}$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{27 \times 10^{-2}}{5000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 107.4295 \times 10^3 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

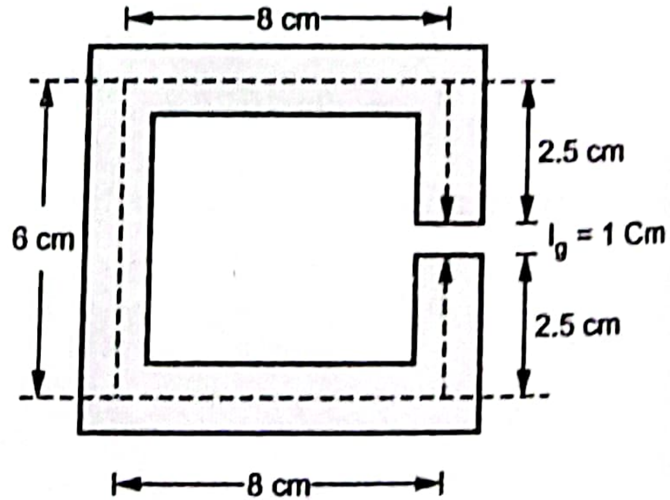


Fig. 1.18.7 (a)

$$= 19.8943 \times 10^{6} \text{ AT/Wb}$$

$$\therefore \quad S_T = S_i + S_g = 20.00173 \times 10^{6} \text{ AT/Wb}$$

But $\quad \phi = \dfrac{\text{m.m.f.}}{S_T}$ i.e. $\quad 0.2 \times 10^{-3} = \dfrac{NI}{20.00173 \times 10^{6}}$

$$\therefore \quad I = \frac{0.2 \times 10^{-3} \times 20.00173 \times 10^{6}}{1000} = \mathbf{4\ A}$$

7 (iii) A coil of 300 turns and of resistance 10Ω is wound uniformly over a steel ring of mean circumference 30 cm and cross-sectional area 9 cm². It is connected to a supply at 20 V D.C. If the relative permeability of the ring is 1500, Find

(i) The magnetising force

(ii) Reluctance

(iii) The m.m.f

(iv) The flux:

given.

$N = 300$

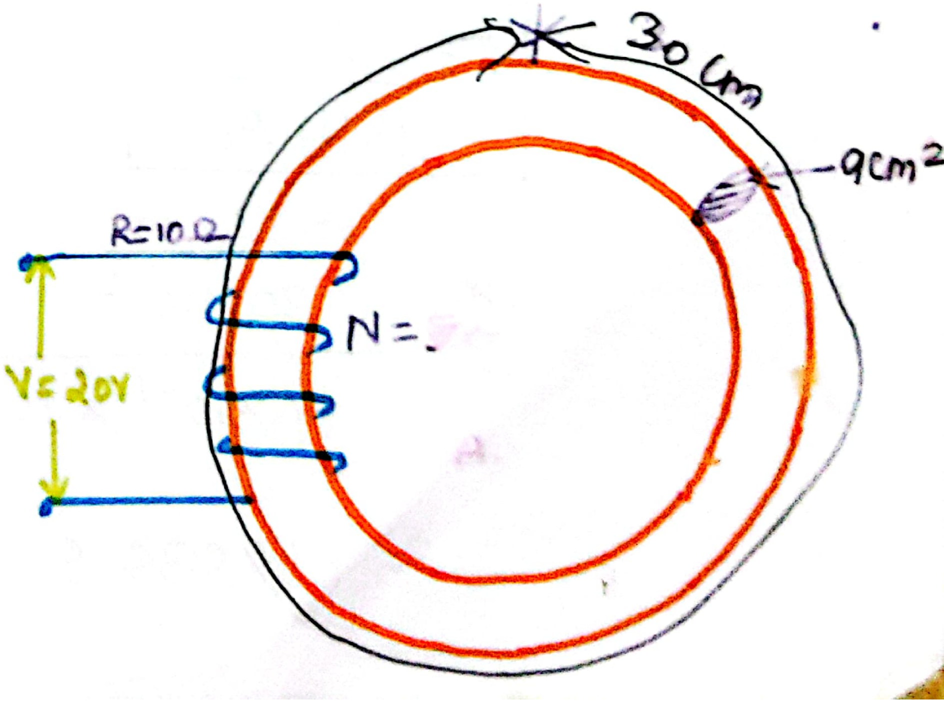
$R = 10\Omega$

$l = 30$ cm

$A_c = 9$ cm²

$V = 20$ V

$\mu_r = 1500$



To Find

(i) H

(ii) S

(iii) M.M.F

(iv) ϕ

Solution.

(i) Magnetising force H

$$\boxed{H = \frac{NI}{l}}$$

$$I = \frac{V}{R}$$

$$I = \frac{20}{10}$$

$$I = 2A$$

$$H = \frac{300 \times 2}{30 \times 10^{-2}}$$

$$\boxed{H = 2000 \ AT/m}$$

(ii) The reluctance, S

$$\boxed{S = \frac{l}{\mu_0 \mu_r A_c}}$$

$$S = \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 1500 \times 9 \times 10^{-4}}$$

$$S = 176838.8 \ AT/Wb$$

(iii) The m.m.f

$$\boxed{m.mf = N.I}$$

$$m.m.f = 300 \times 2$$

$$\boxed{m.mf = 600\ AT}$$

(iv) The flux (ϕ)

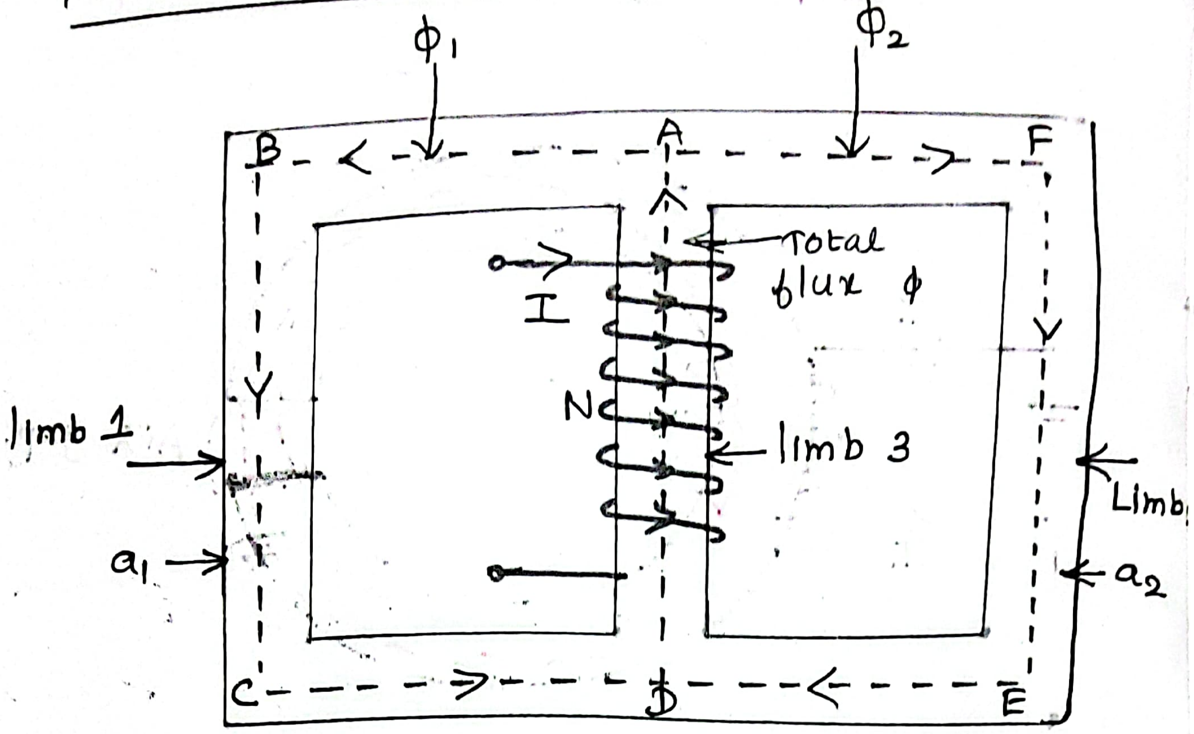
$$\phi = \frac{m.mf}{reluctance}$$

$$\phi = \frac{600}{176838.8}$$

$$\phi = 3.392 \times 10^{-3}\ Wb.$$

$$\boxed{\phi = 3.392\ mWb}$$

PARALLEL MAGNETIC CIRCUITS.



The mean length of path ABCD $= l_1$ m

The mean length of path AFED $= l_2$ m

The mean length of AD $= l_3$ m

The reluctance of ABCD $= S_1$

The reluctance of AFED $= S_2$.

The reluctance of AD $= S_3$.

Total M.MF $=$ NI

$$S_1 = \frac{l_1}{\mu a_1} \; ; \quad S_2 = \frac{l_2}{\mu a_2} \; ; \quad S_3 = \frac{l_3}{\mu a_3}.$$

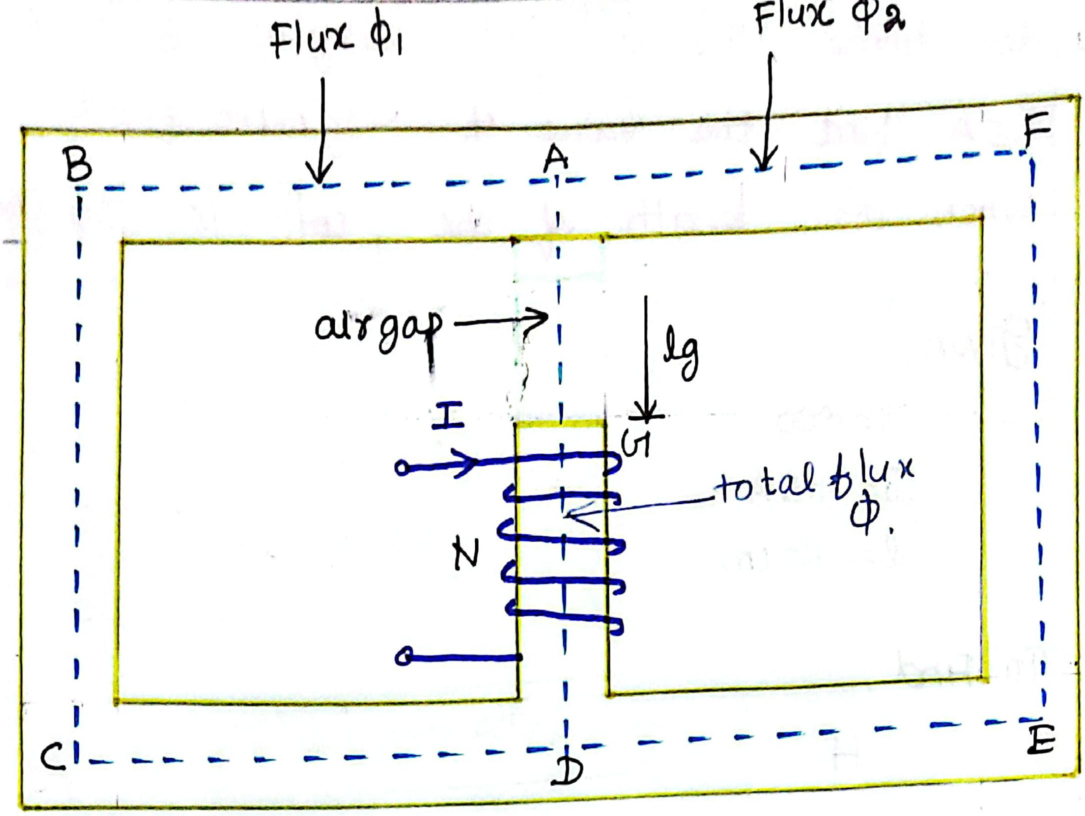
Total $NI = (NI)_{AD} + (NI)_{ABCD}$ or $(NI)_{AFED}$.

For path ABCDA, $NI = \phi_1 S_1 + \phi S_3$

For path ~~AB~~ AFEDA, $NI = \phi_2 S_2 + \phi S_3$

Total MMF = MMf by limb (excited) + mmf by any one but limb

PARALLEL MAGNETIC CIRCUIT WITH AIR GAP



The reluctance of the central limb is now :

$$S_c = S_3 + S_g$$

$$S_c = \frac{l_3}{\mu a_3} + \frac{l_g}{\mu_0 a_3}$$

$$(MMF)_{AD} = (mmf)_{GD} + (mmf)_{GA}$$

$$\boxed{\text{Total } NI = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD} \text{ or } (NI)_{AFED}.}$$

Example 1.18.7 *A cast steel structure is made of a rod of square section 2.5 cm × 2.5 cm as shown in the Fig. 1.18.14. What is the current that should be passed in a 500 turn coil on the left limb so that a flux of 2.5 mWb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.*

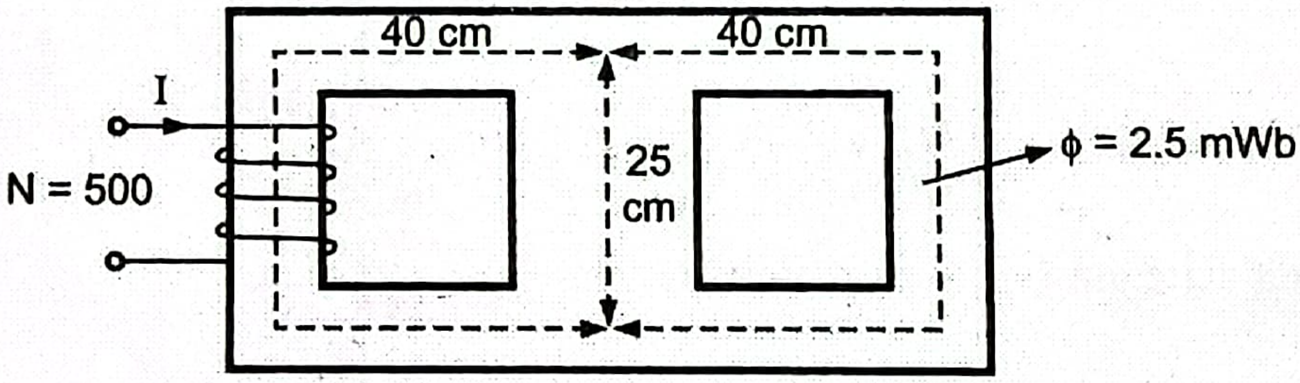


Fig. 1.18.14

Solution :

This is parallel magnetic circuit. Its electrical equivalent is shown in the Fig. 1.18.14 (a).

The total flux produced gets distributed into two parts having reluctances S_1 and S_2.

S_1 = Reluctance of centre limb

S_2 = Reluctance of right side

$$S_1 = \frac{l_1}{\mu_0 \mu_r a_1} = \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 750 \times 2.5 \times 2.5 \times 10^{-4}}$$

$$= 424.413 \times 10^3 \text{ AT/Wb}$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r a_1} = \frac{40 \times 10^{-2}}{4\pi \times 10^{-7} \times 750 \times 2.5 \times 2.5 \times 10^{-4}}$$

$$= 679.061 \times 10^3 \text{ AT/Wb}$$

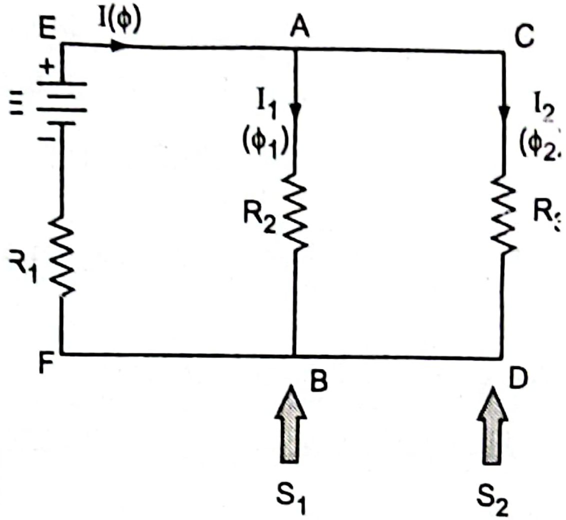


Fig. 1.18.14 (a)

Key Point *For parallel branches, m.m.f. remains same.*

For branch AB and CD, m.m.f. is same.

$\therefore$ $$\text{m.m.f.} = \phi_1 S_1 = \phi_2 S_2$$

And $$\phi_2 = 2.5 \text{ mWb}$$ ... Given

$\therefore$ $$\phi_1 = \frac{\phi_2 S_2}{S_1} = \frac{2.5 \times 10^{-3} \times 679.061 \times 10^3}{424.413 \times 10^3} = 4 \text{ mWb}$$

$\therefore$ $$\phi = \phi_1 + \phi_2 = 2.5 + 4 = 6.5 \text{ mWb}$$

Total m.m.f. required is sum of the m.m.f. required for AEFB and that for either central or side limb.

$$S_{AEFB} = S_2 = 679.061 \times 10^3 \text{ AT/Wb}$$

$\therefore$ m.m.f. for AEFB $= S_{AEFB} \times \phi = 679.061 \times 10^3 \times 6.5 \times 10^{-3} = 4413.8965$ AT

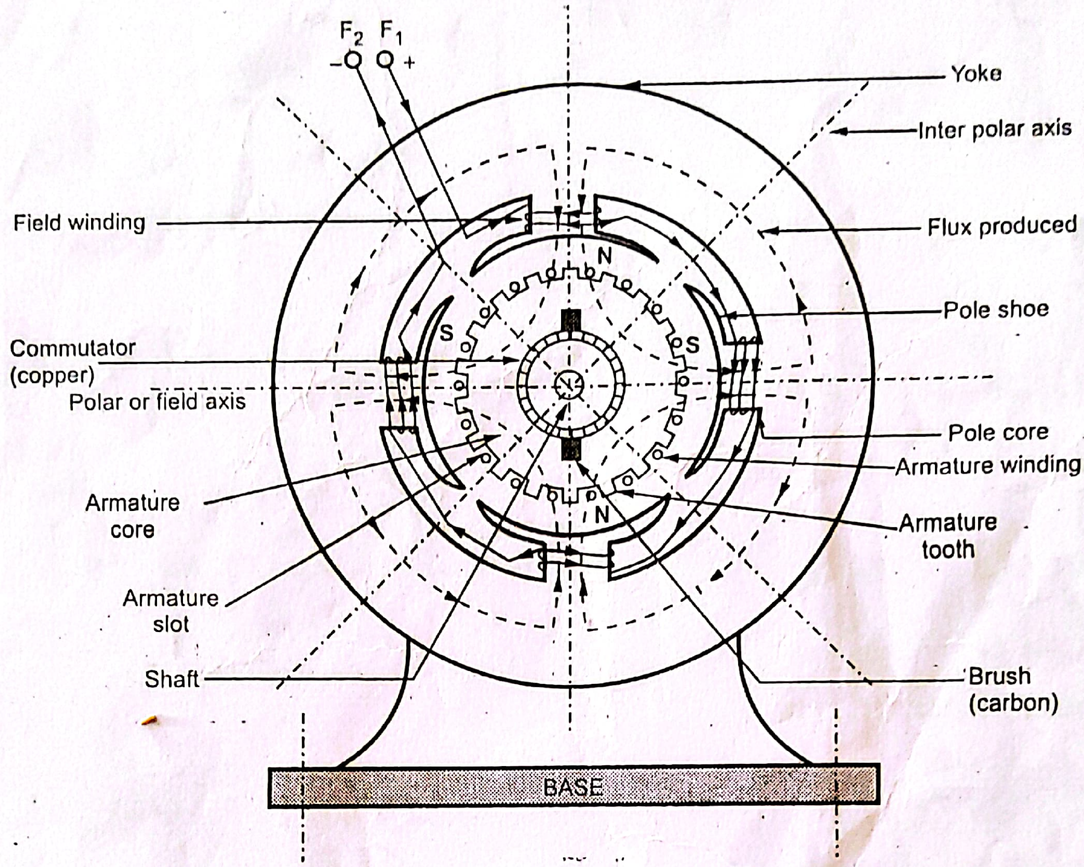
$\therefore$ $$\text{Total m.m.f.} = 4413.8965 + \phi_1 S_1$$

$$= 4413.8965 + 4 \times 10^{-3} \times 424.413 \times 10^3 = 6111.548 \text{ AT}$$

But $$NI = \text{Total m.m.f.}$$

$\therefore$ $$I = \frac{6111.548}{500} = 12.223 \text{ A}$$

CONSTRUCTION OF A D.C MACHINE



1) YOKE

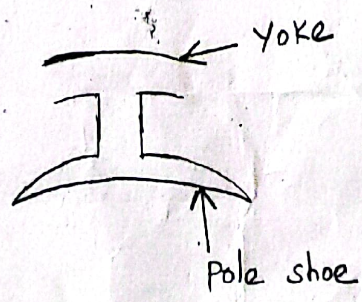
FUNCTIONS :

1. It provides mechanical support to the whole machine.
2. It provides path for pole flux.

CHOICE OF MATERIAL :

→ It is made by cast steel or fabricated steel.

2) POLES



Functions:-

1. Pole carries field Winding and it produces flux.
2. pole shoe spreads out the flux in the air gap.

Choice of Material:

→ It is made by cast iron or cast steel.

3) Field Winding

Functions

1. It is wound on the pole core. It is excited by DC supply.

Choice of material:

→ Copper.

4) INTER POLES:

Functions

1. To reduce armature reaction.
2. It is placed between two poles.

Choice of material:

→ made up of solid steel.

5) BRUSHES

Functions:

1. It provides connection between the revolving armature and the external circuit.
2. Brushes are placed in the brush holder. A spring presses it against the commutator surface.

Choice of material:

→ made up of carbon, Graphite, carbon Graphite, metal graphite etc..

6) ARMATURE CORE:

Functions

→ Rotating part
→ has slots and slot carries armature coil.

Choice of material:

→ 0.35 to 0.5 mm thick laminations of silicon steel.

7) ARMATURE WINDING:

→ Two types of armature winding
(i) lap winding
(ii) Wave Winding.

→ It placed In the slots of armature core.

Choice of material:

→ Copper.

8) COMMUTATOR

1. It is cylinderical in structure.
2. It is made up of wedge shaped copper segments.

Functions:

Converts AC to DC.

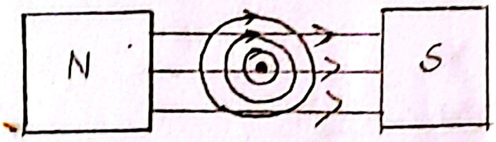
9) SHAFT

commutator, Armature, bearing are mounted on shaft.

DC MOTOR - PRINCIPLE OF OPERATION

Principle:

"Whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force."



Here there are 2 fluxes.

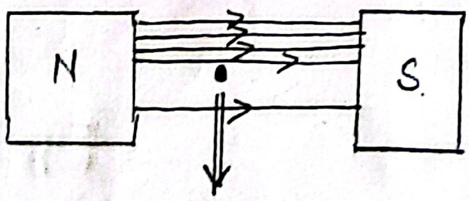
✓ flux produced by permanent magnet called main flux.

✓ flux produced by current carrying conductor called armature flux

From fig.

* on one side of the conductor, both fluxes are in same direction.
* on the other side of the conductor, both the fluxes are in opposite direction.

✓ Thus above conductor the flux is strengthened and below the conductor the field is weakend.



→ Thus a mechanical force on the conductor is produced from high flux area to low flux area.

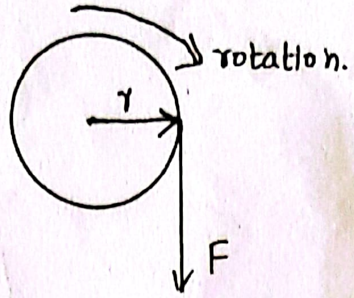
→ The magnitude of force, $\boxed{F = BIl}$ Newton

B → Flux density,
I → current
l → length of Conductor

TORQUE EQUATION OF DC MOTOR

→ Torque is nothing but turning or twisting force about axis.

→ Torque = Force × radius.



Torque, $T = F \times r$ ——— (1)

✓ Work done = Force × distance.

✓ For one revolution, Work done, $W = F \times 2\pi r$ ——— (2)

✓ Power = $\frac{\text{Work done}}{\text{Time}}$

For one revolution, power = $\frac{F \times 2\pi r}{\frac{60}{N}}$

$$P = \frac{F \times 2\pi r \times N}{60} \quad \text{——— (3)}$$

Angular velocity $\omega = \frac{2\pi N}{60}$ ——— (4)

sub (4) in (3)

~~Comparing~~ (3) & (4)

$$(3) \Rightarrow \quad P = F \times r \times \omega$$

$$\Rightarrow \boxed{P = T\omega} \quad \text{——— (5)}$$

Electrical power,

$$P = E_b I_a \quad \text{——— (6)}.$$

Equating (5) & (6)

$E_b I_a$

$$E_b I_a = T\omega$$

$$\frac{\phi P N Z}{60 A} I_a = T \cdot \frac{2\pi N}{60}$$

$$T = \frac{\cancel{60}\ \phi P \cancel{N} Z \cdot I_a}{\cancel{60} A\ 2\pi \cancel{N}}$$

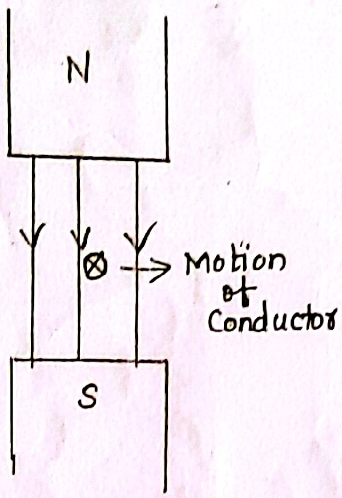
$$\boxed{T = \frac{1}{2\pi} \phi I_a \frac{PZ}{A}}$$

DC GENERATOR - PRINCIPLE OF OPERATION

Principle:

"Whenever flux linking with a conductor changes, an emf is induced in the conductor".

$$\boxed{E = N.\frac{d\phi}{dt}}$$



→ Here Magnitic field is provided by two magnetic poles.

→ Magnetic field is from North to south pole.

✓ A Mechanical force is applied to the conductor such that the conductor move from left to right with velocity 'V' m/sec. Due to this movement flux linking the conductor changes and hence an emf induced in the conductor which is given by,

$$\boxed{E = Blv}$$

B → flux density
l → Length of the conductor
V → velocity of the conductor ⊥r to direction of flux.

✓ If the conductor moves at an angle 'θ' degrees with respect to magnetic field then, emf induced in the conductor is given by

$$\boxed{E = Blv \sin\theta}$$

EMF EQUATION OF DC GENERATOR

let

- $P \rightarrow$ No. of poles of generator
- $\phi \rightarrow$ flux per pole.
- $N \rightarrow$ speed of armature in r.p.m
- $Z \rightarrow$ No of armature conductors.
- $A \rightarrow$ No. of parallel paths.

✓ According to farday's law,

EMf induced, $E = \dfrac{d\phi}{dt}$ ——— (1)

✓ In one revolution,

* Flux $= P\phi$ ——— (2)
* Time for one revolution $= \dfrac{60}{N}$ ——— (3)

sub (2) & (3) in (1)

$$E = \dfrac{P\phi}{\frac{60}{N}}$$

$$E = \dfrac{NP\phi}{60}$$

There are Z conductors with A parallel paths

$$\boxed{E = \dfrac{NP\phi}{60}\left(\dfrac{Z}{A}\right)}$$

Back emf (E_b)

2 mark

once the armature start rotating as a motor, the main flux gets cut by the armature winding and an emf is induced in the armature. This emf opposes the applied voltage and is called Back emf.

$$\boxed{E_b = \dfrac{\phi PNZ}{60A}}$$

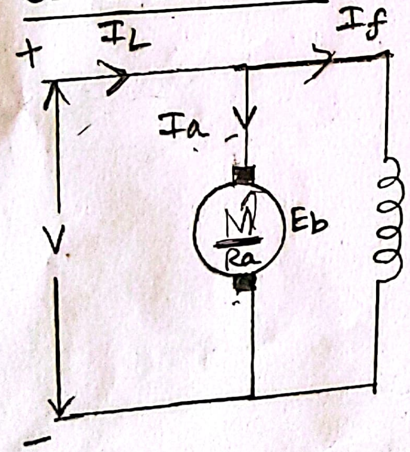
Difference between Lap and wave winding.

Lap Winding	Wave Winding.
1. No of parallel path is equal to No of poles. $A = P$	No of parallel path $= 2$
2. preferrable for high current, low voltage.	preferrable for high voltage low current.

Types of DC MOTOR

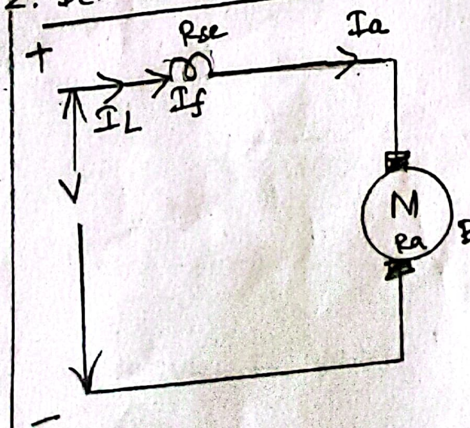
1. DC shunt Motor
2. DC series Motor
3. DC Compound Motor → long shunt, short shunt.

1. DC shunt Motor



* The armature and field winding are connected in parallel.
* $R_a \rightarrow$ Armature Resistance
* current Equation, $\boxed{I_L = I_a + I_f}$
* Voltage Equation, $\boxed{V = E_b + I_a R_a}$

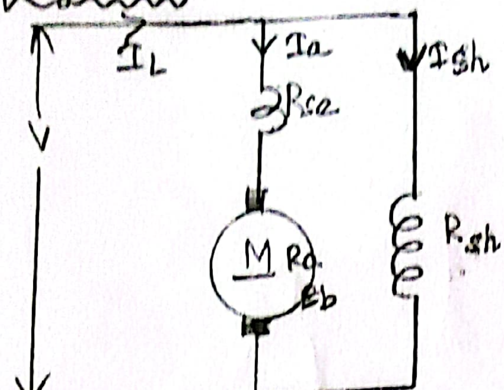
2. DC series Motor



* The armature and field windings are connected in series.
* current equation, $\boxed{I_L = I_f = I_a}$
* voltage equation, $\boxed{V = E_b + I_a R_{se} + I_a R_a}$

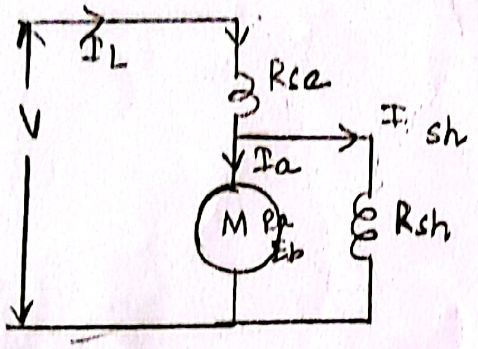
* DC Compond Generator:

Long shunt



* It consists of both shunt and series field winding.
* current equation, $\boxed{I_L = I_a + I_{sh}}$
* voltage equation, $\boxed{V = E_b + I_a R_a + I_a R_{se}}$

Short shunt



* Current Equation $\boxed{I_L = I_a + I_{sh}}$
* voltage Equation, $\boxed{V = E_b + I_a R_a + I_L R_{se}}$

CHARACTERISTICS OF DC MOTOR

(1) speed - Armature current characteristics.
(2) Torque - Armature current characteristics.
(3) speed torque characteristics.

CHARACTERISTICS OF DC SHUNT MOTOR

(1) speed - Armature current characteristics (N Vs Ia)

For DC Motor,

$$N \propto \frac{E_b}{\phi}$$

For DC shunt motor,

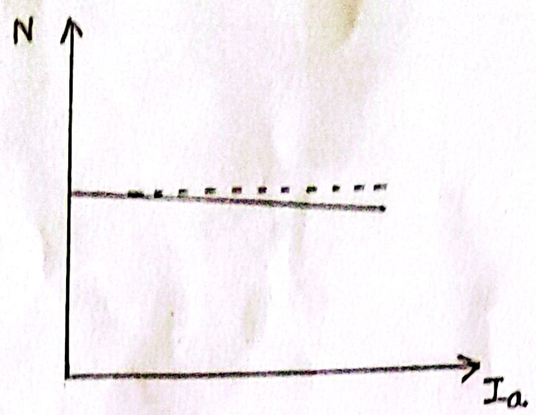
$$\phi \rightarrow \text{constant}$$

$$\therefore N \propto E_b$$

$$\Rightarrow N \propto V - I_a R_a$$

✓ when Load increased 'Ia' increased. But Ra is very small. So $I_a R_a$ is very smal.

✓ Hence 'N' is almost constant for constant supply voltage.

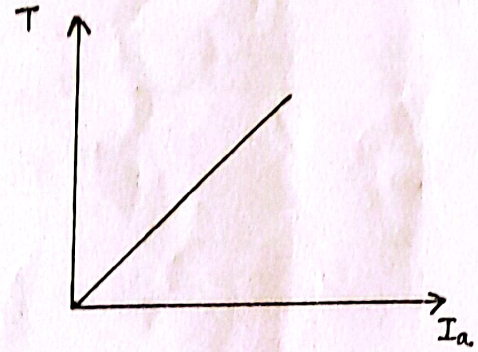


(2) Torque - Armature current characteristics (T vs Ia) ;

$$T \propto \phi I_a$$

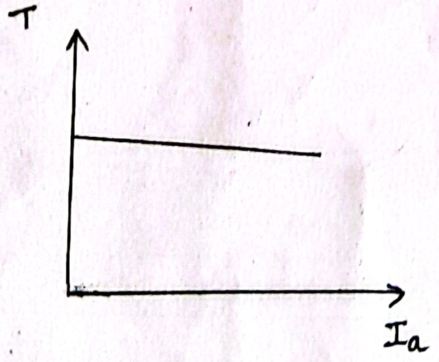
For shunt motor, $\phi \rightarrow$ constant

Hence, $T \propto I_a$



(3) Speed - Torque characteristics (N vs T) :-

→ It is also called mechanical characteristics.



CHARACTERISTICS OF DC SERIES MOTOR

(1) Speed Armature current characteristics (N vs Ia) :

For DC Motor

$$N \propto \frac{E_b}{\phi}$$

For DC series Motor,

$$I_L = I_a = I_f$$

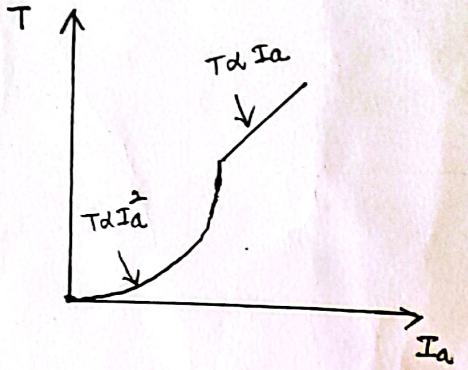
(2) Torque - Armature current characteristics (T vs I_a).

For DC Motor, $T \propto \phi I_a$

For DC series motor, $\phi \propto I_a$

$\Rightarrow T \propto I_a^2$ (before saturation)

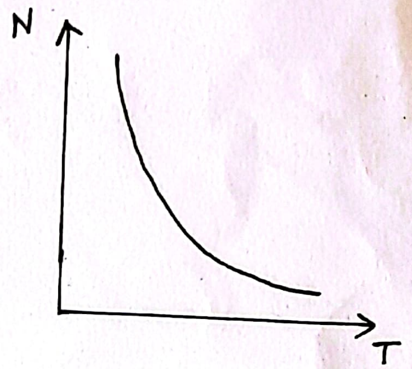
$\Rightarrow T \propto I_a$ (After saturation)



(3) Speed - Torque characteristics (N vs T)

For DC series Motor, $T \propto I_a^2$ & $N \propto \frac{1}{I_a}$

$\Rightarrow N \propto \frac{1}{\sqrt{T}}$



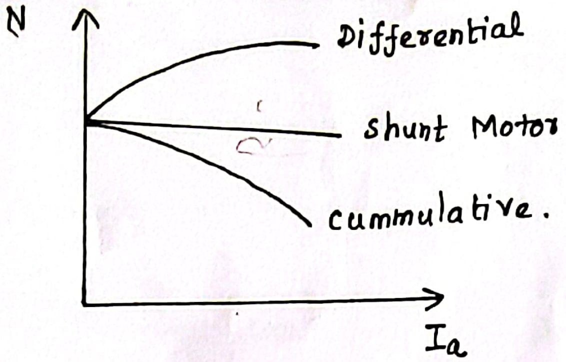
CHARACTERISTICS OF COMPOUND MOTOR

Speed - Armature current characteristics (N vs I_a) :-

For DC Motor, $N \propto \frac{E_b}{\phi}$

✓ In cumulative compound motor, when load increases, I_a increases, flux increases. so speed decreases.

✓ In differential compound motor, when load increases, I_a increases, flux decreases. so speed increases

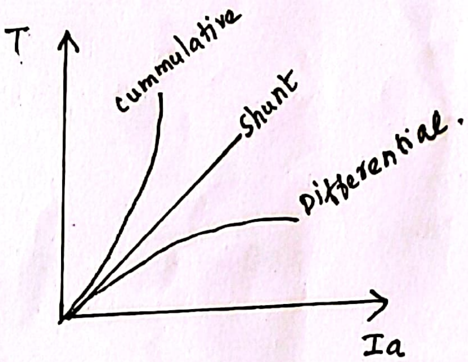


2. Torque - Armature current characteristics. (T Vs I_a)

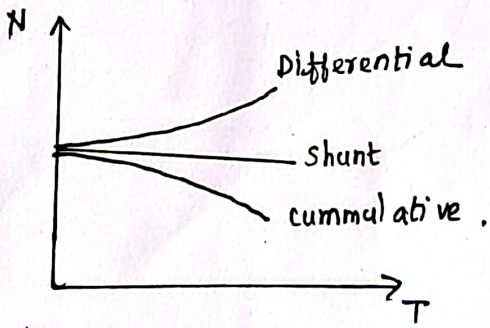
For DC motor, $T \propto \phi I_a$

✓ For cummulative compound motor ϕ_{sh} & ϕ_{se} are additive, so torque increas.

✓ For Differential compound motor. ϕ_{sh} & ϕ_{se} oppose each other, so torque decreases.



3. Speed - Torque characteristics (N vs T)



Mark

APPLICATIONS OF DC MOTOR

1. DC series Motor

Cranes, Electric train, Hoist, Elevator.

2. DC shunt Motor

Lathe machine, Fan, centrifugal pump, Machine tools.

3. DC Compound Motor

Printing machine, conveyors, compressors.

P→ No. of

Why series motor is never started on No load :

speed equation $N \propto \frac{E_b}{\phi}$

$N \propto \frac{1}{\phi}$ [Eb is almost constant]

$\phi \propto I_a$ [For series motor]

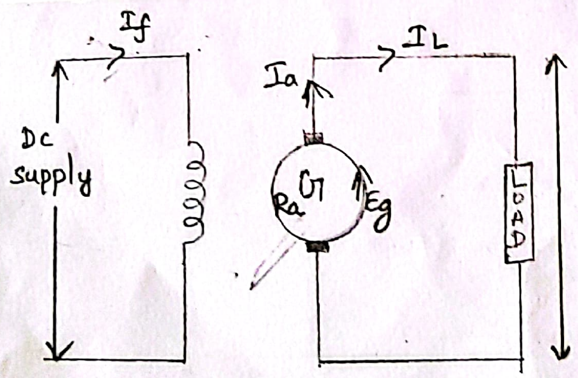
2mark

* At less load or no load I_a is very small. It means flux is very small.
* The motor tries to run at dangerously high speed which may damage the motor mechanically.

Types of DC Generator

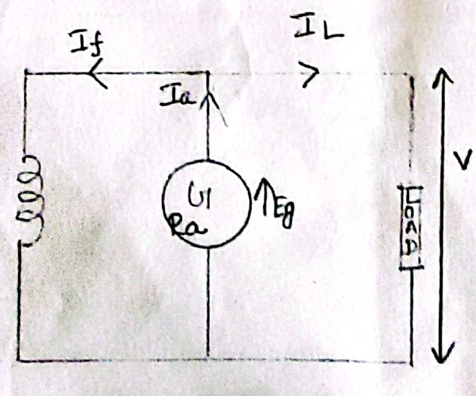
1. Seperately Excited Generator
2. Self Excited Generator.
   (i) shunt Generator
   (ii) series Generator
   (iii) Compound Generator.

Seperately Excited DC Generator



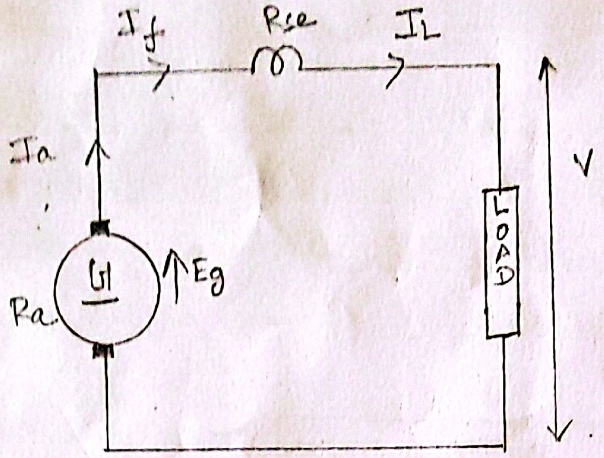
$$E_g = V + I_a R_a + V_{brush}$$

shunt Generator



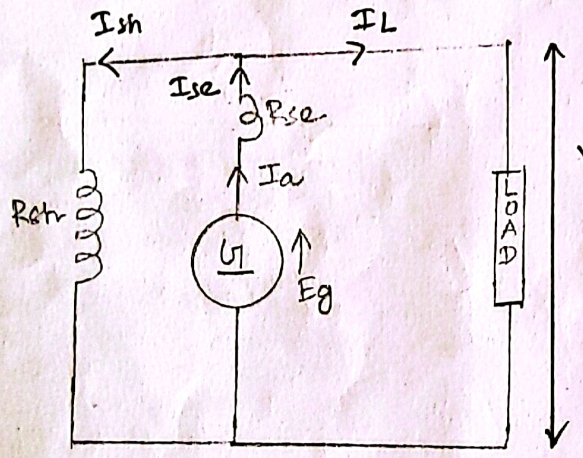
$$E_g = V + I_a R_a$$
$$I_a = I_L + I_{sh}$$

SERIES GENERATOR



$$\boxed{\begin{aligned} I_a &= I_f = I_L \\ E_g &= V + I_a R_a + I_f R_{se} \end{aligned}}$$

COMPOUND GENERATOR



$$\boxed{\begin{aligned} I_a &= I_{se} = I_{sh} + I_L \\ E_g &= V + I_a R_a + I_{se} R_{se} \end{aligned}}$$

TRANSFORMER

What is Transformer :-

2 mark

Transformer is a static device, by means of which an electrical power is transformed from one circuit to another with the desired change in voltage and current, with out any change in frequency.

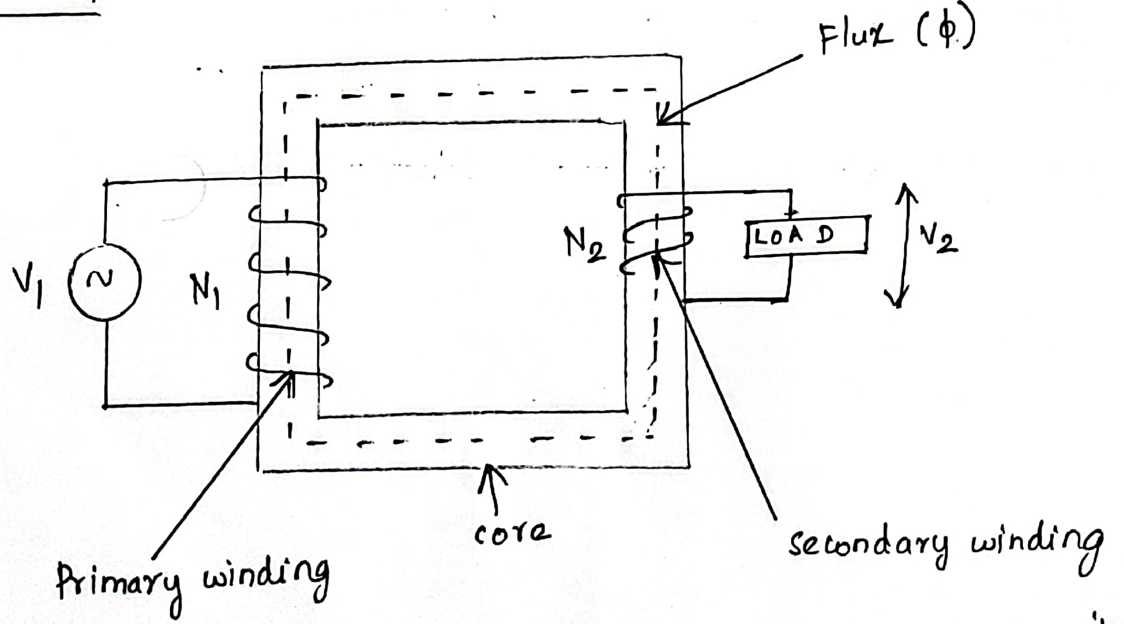
PRINCIPLE OF OPERATION OF TRANSFORMER

The transformer operates on the principle of "mutual Induction"

2 mark

It is the phenomenon in which a change of current in one coil causes an induced emf in another coil placed near to the first coil.

Explanation :-



→ When primary winding is connected to an AC source, it will produce alternating flux.

→ This flux pass through the core and link with secondary winding.

→ Thus due to this change in flux (∵ alternating flux) link with secondary coil induces an emf.

→ The frequency of the induced emf in the secondary winding is same as the frequency of the supply voltage.

→ In a transformer, if the number of turns in the secondary is less than the primary winding, it is called step-down transformer.

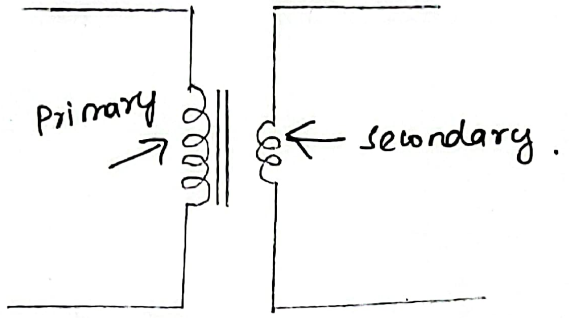


Fig. step-down transformer.

→ In a transformer, if the number of turns in the primary is less than the secondary winding, it is called step-up transformer.

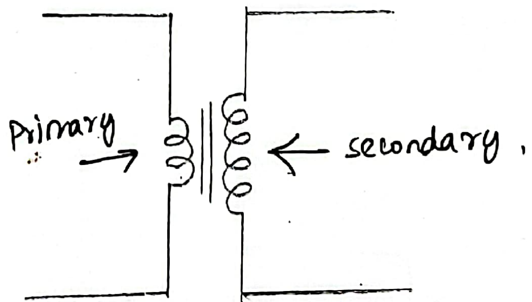


Fig. step up transformer.

Transformation Ratio. (K):-

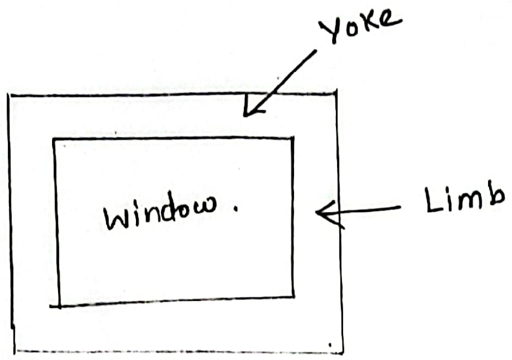
2 mark

$$\boxed{K = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}}$$

TRANSFORMER CONSTRUCTION

The basic parts of transformer are
1. Magnetic core
2. Primary and secondary windings.

MAGNETIC CORE



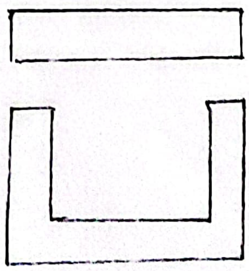
→ Magnetic core is a stack of thin silicon-steel laminations (to reduce hysteresis and eddy current loss) of about 0.35 mm thick.

→ The vertical portion of the core is called Limb.

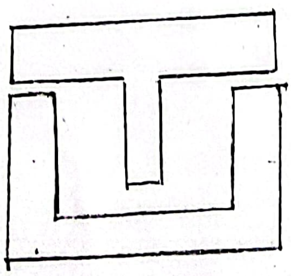
→ The top and bottom horizontal portion of the core is called Yoke.

→ The space enclosed by limb and Yoke is called window.

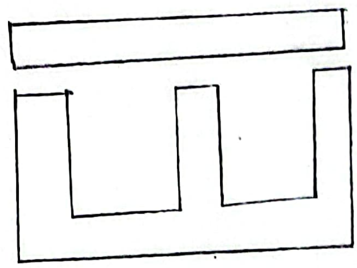
The core lamination have following shapes,



U & I sections



U & T sections



E & I sections.

WINDINGS

There are two types of transformers based on winding wound.

1. core type
2. shell type.

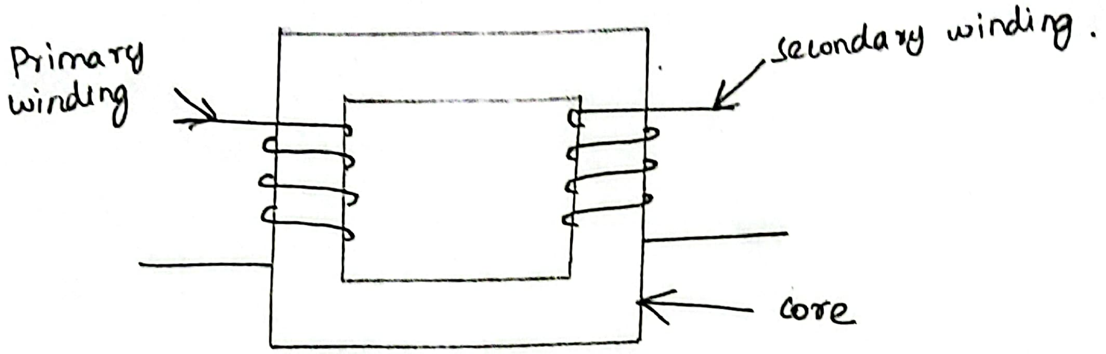
CORE TYPE

→ In core type transformer the winding surround the considerable part of the core.

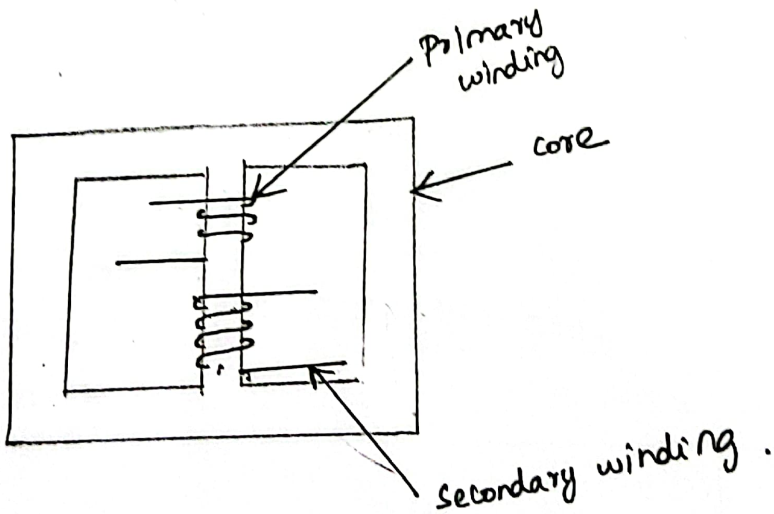
→ It has two limbs.

→ cylinderical coils are used.

→ Preferred for low voltage transformers.



SHELL TYPE :-



→ In shell type core encircles most part of the winding.

→ It has 3 limbs

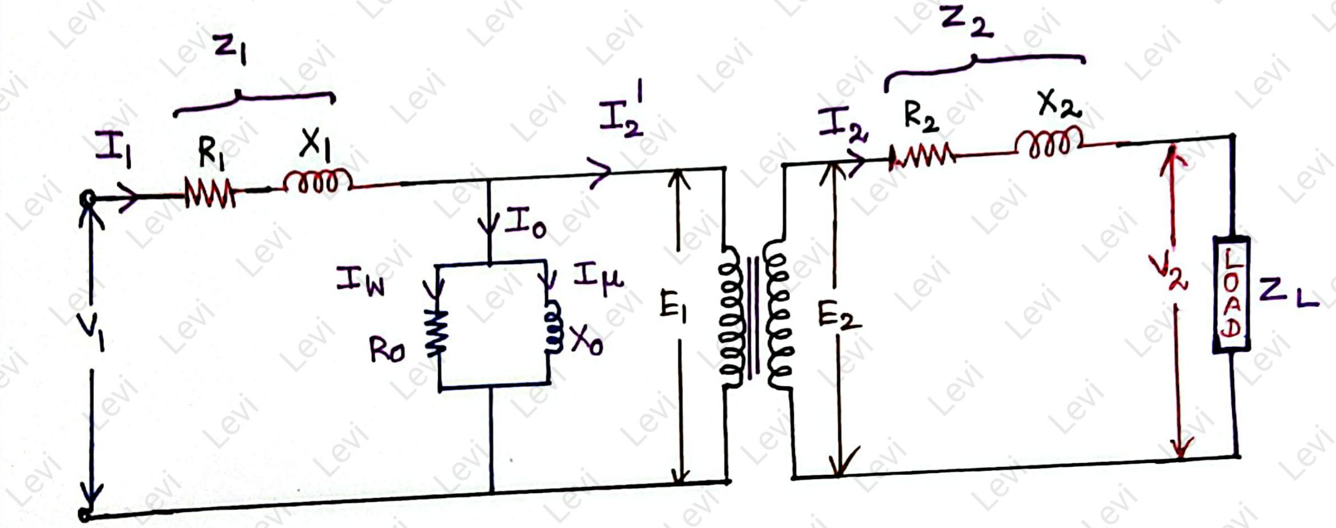
→ The coils are generally multilayer disc type.

→ Preferred for high voltage transformers.

EQUIVALENT CIRCUIT OF A TRANSFORMER

If any electrical device is to be analyzed and investigated for further modifications, the use of an appropriate equivalent circuit is necessary.

Equivalent circuit is simply a circuit representation of the equation describing the performance of the device.



In the equivalent circuit,

$R_1 \rightarrow$ Primary winding resistance
$X_1 \rightarrow$ Primary leakage reactance.
$R_1 + jX_1 = Z_1 \Rightarrow$ Impedance of primary winding.

$R_2 \rightarrow$ Secondary winding resistance
$X_2 \rightarrow$ Secondary leakage reactance
$R_2 + jX_2 = Z_2 \Rightarrow$ Impedance of secondary winding.

The components of primary current (I_1) are.

I_1:
- $I_2' = K I_2$ (load component of primary current)
- I_0 (No load primary current)
  - I_W (Working component)
  - I_μ (Magnetising component)

$E_1 \rightarrow$ Induced emf in primary winding.

$E_2 \rightarrow$ Induced emf in Secondary winding.

$Z_L \rightarrow$ Load Impedance.

$V_2 \rightarrow$ output Voltage.

$K \rightarrow$ Transformation Ratio.

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary.

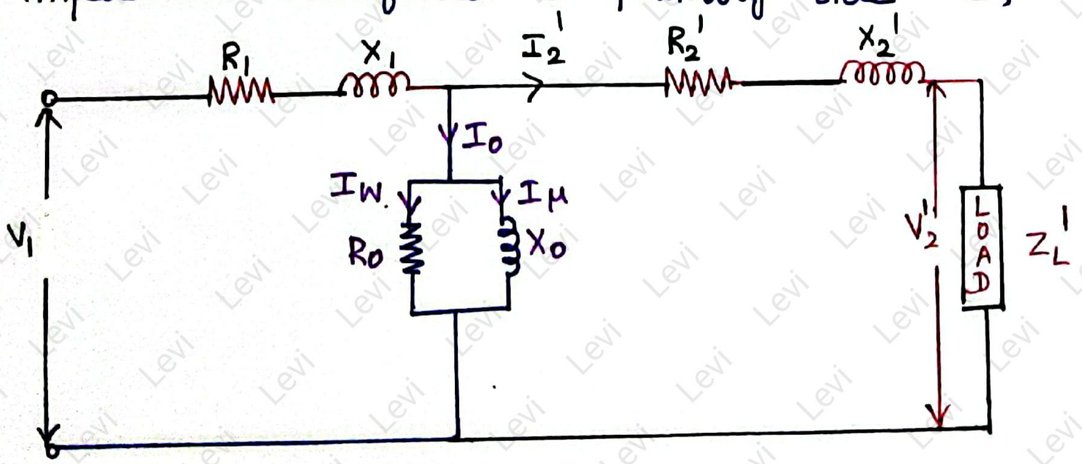
→ When secondary parameters are referred to primary,

- ✓ Resistances and Reactances are divided by K^2
- ✓ Voltages are divided by K,
- ✓ currents are multiplied by K

$$R_2' = \frac{R_2}{K^2} \quad ; \quad X_2' = \frac{X_2}{K^2} \quad ; \quad Z_L' = \frac{Z_L}{K^2}$$

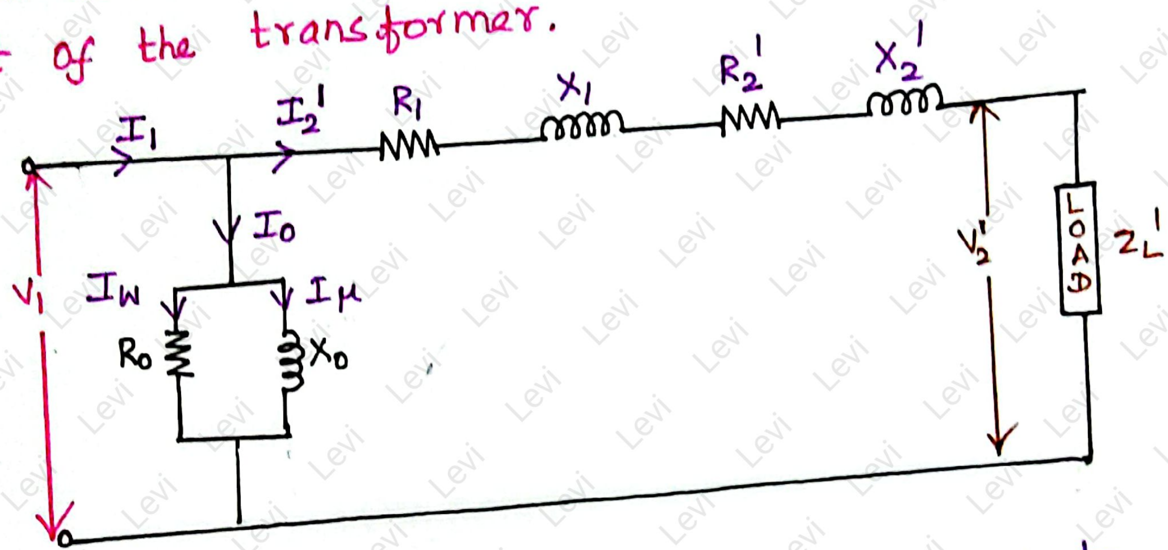
$$I_2' = KI_2 \quad ; \quad V_2' = \frac{V_2}{K}$$

The exact equivalent circuit of the transformer with all impedances transferred to primary side is,



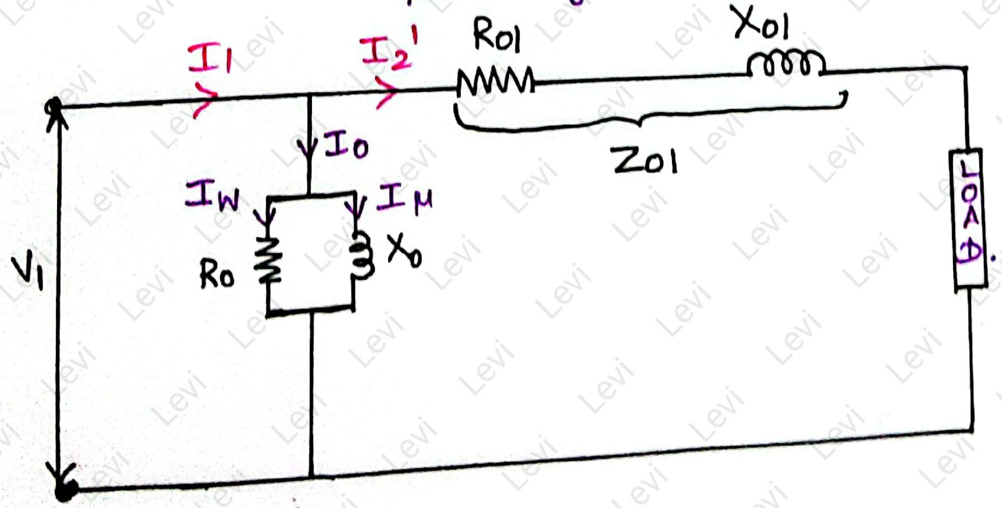
- It is seen that E_1 differs from V_1 by a very small amount $(E_1 \approx V_1)$

- Moreover, the no-load current (I_0) is only a small fraction of full load primary current. So that I_2' is practically equal to I_1. So shift the no-load branch to the left position of the circuit. This circuit is known as approximate equivalent circuit of the transformer.



$R_1 + R_2' \Rightarrow R_{01} \Rightarrow$ Equivalent resistance referred to primary.

$X_1 + X_2' \Rightarrow X_{01} \Rightarrow$ Equivalent reactance referred to primary.



$$\boxed{R_0 = \frac{V_1}{I_W} \qquad X_0 = \frac{V_1}{I_\mu}}$$

EMF Equation of a Transformer

Let,

$N_1 \rightarrow$ Number of turns in the primary winding.

$N_2 \rightarrow$ Number of turns in the secondary winding.

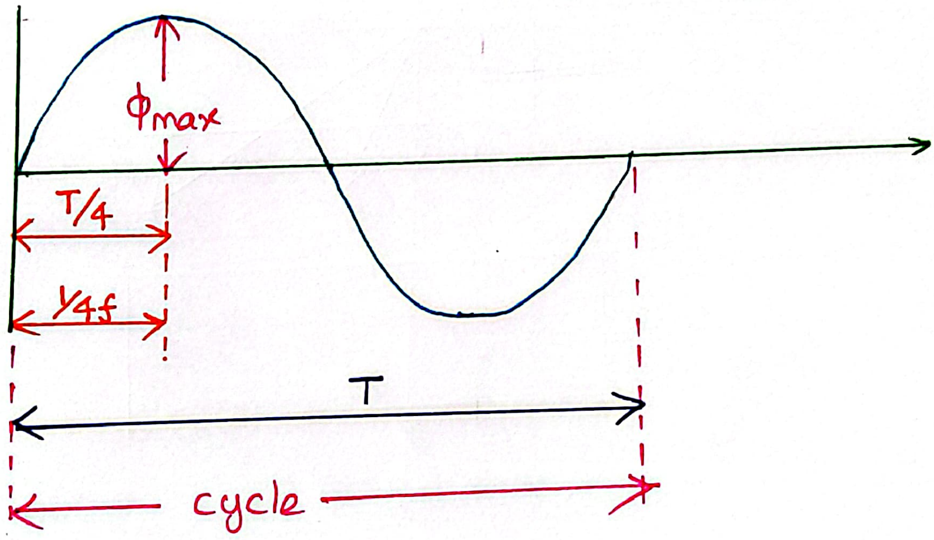
$\phi_m \rightarrow$ Maximum flux in the core, Wb.

$$\phi_m = B_m \times A$$

$$= \text{Maximum flux density} \times \text{core cross section Area.}$$

$f \rightarrow$ frequency of A.C input, Hz

The applied Voltage is AC (sinusoidal). so the flux established is also sinusoidal.



According to Faraday's law,

$$\text{Average EMF per turn} = \frac{d\phi}{dt} \Rightarrow \frac{\text{change in Flux}}{\text{Time}}$$

consider $\frac{1}{4}$th cycle of flux.

$$\frac{d\phi}{dt} = \frac{\phi_m - 0}{\frac{1}{4f}}$$

$$\text{Average EMF per turn} = 4f\phi_m$$

For sinusoidal quantity,

$$\text{Form Factor} = \frac{\text{R.M.S. Value}}{\text{Average Value}} = 1.11$$

⇒

RMS value = 1.11 × Average value

∴ RMS value of EMF/turn $= 1.11 \times 4 f \phi_m$

$= 4.44 f \phi_m$.

Now, RMS value of induced EMF in the whole of primary winding is,

$$\boxed{E_1 = 4.44 f \phi_m N_1}$$

Similarly, RMS value of induced EMF in the whole of secondary winding is,

$$\boxed{E_2 = 4.44 f \phi_m N_2}$$

Voltage Transformation Ratio (K)

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$

PHASOR DIAGRAMS FOR TRANSFORMER ON LOAD

UNITY POWER FACTOR LOAD, $\cos\phi_2 = 1$

→ For unity power factor V_2 and I_2 are inphase.

steps to draw phasor diagram :-

1. Consider flux $\vec{\phi}$ as reference.
2. $\vec{E_1}$ lag $\vec{\phi}$ by $90°$. Reverse $\vec{E_1}$ to get $-\vec{E_1}$
3. $\vec{E_1}$ and $\vec{E_2}$ are inphase.
4. Assume $\vec{V_2}$ in particular direction.
5. I_2 is (inphase or lag or lead) (type of load)
6. Add $\overrightarrow{I_2R_2}$ and $\overrightarrow{I_2X_2}$ to $\vec{V_2}$ to get $\vec{E_2}$
7. Reverse $\vec{I_2}$ to get $\vec{I_2'}$
8. Add $\vec{I_0}$ and $\vec{I_2'}$ to get $\vec{I_1}$
9. Add $\overrightarrow{I_1R_1}$ and $\overrightarrow{I_1X_1}$ to $-\vec{E_1}$ to get $\vec{V_1}$

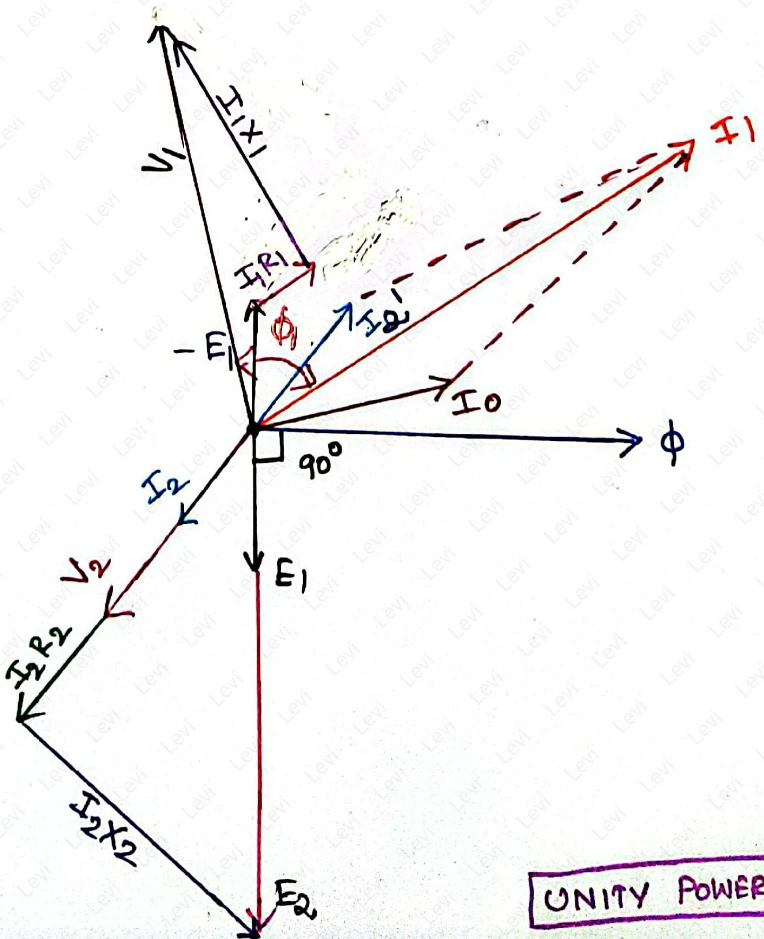
Note:

$I_2R_2 \parallel I_2$

$I_2X_2 \perp r\ I_2R_2$

$I_1R_1 \parallel I_1$

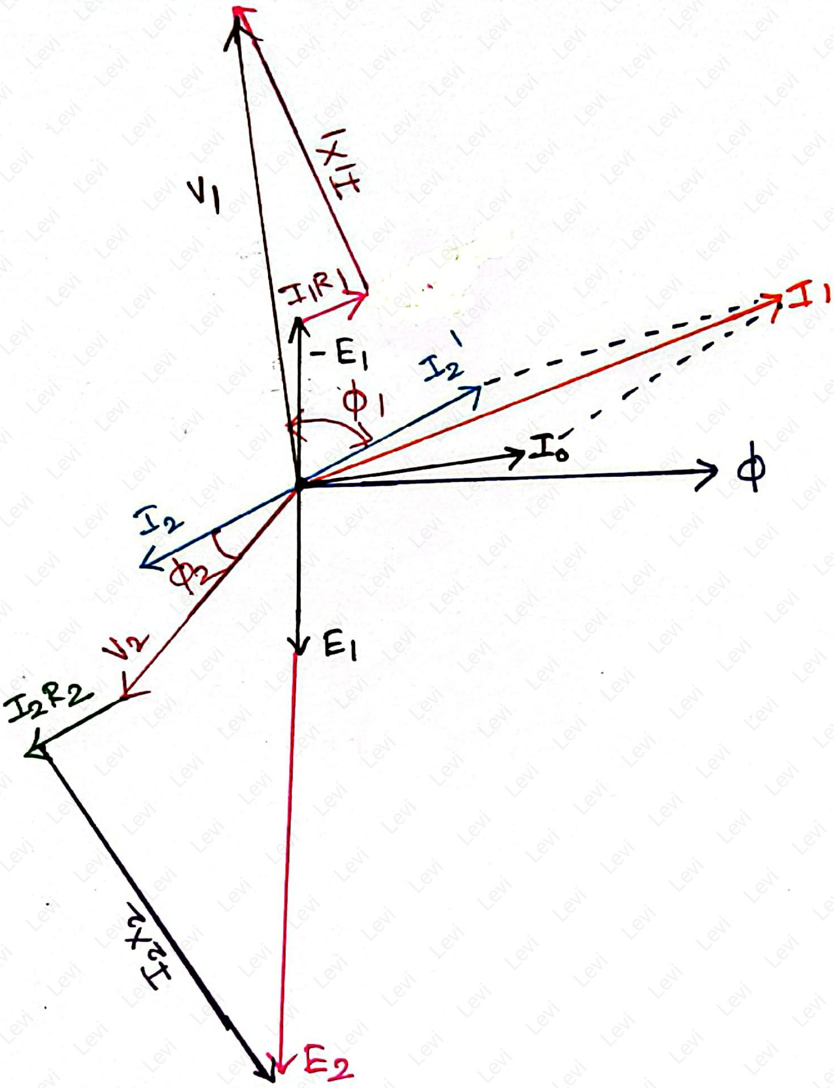
$I_1X_1 \perp r\ I_1R_1$



UNITY POWER FACTOR

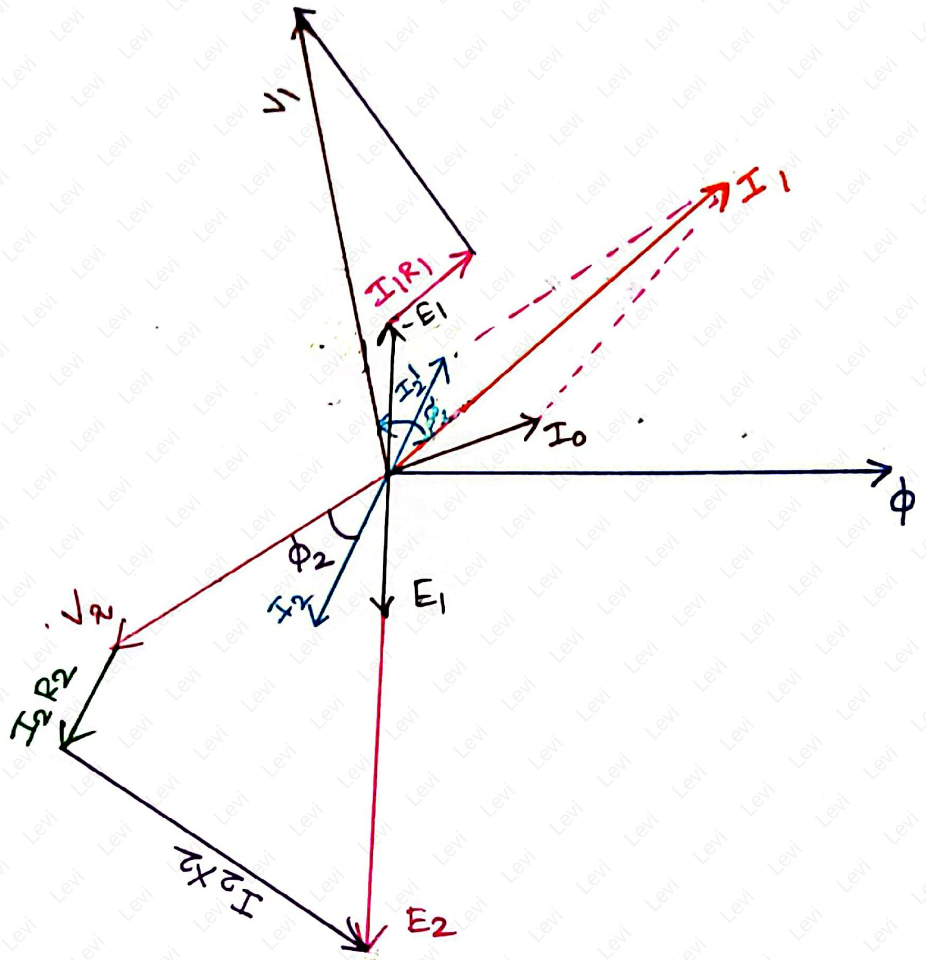
LAGGING POWER FACTOR LOAD

Here, the current I_2 lags V_2 by an angle ϕ_2.



LEADING POWER FACTOR LOAD

Here, current leads by an angle ϕ_2



VOLTAGE REGULATION OF TRANSFORMER

→ A transformer should have a small value of voltage regulation. (ie. good voltage regulation).

→ For a transformer of large voltage regulation (Poor regulation) the secondary voltage will fall with the increase in load. This has a bad effect on the operation of fluorescent tubes, T.V, refrigerator, motors etc..

→ Voltage regulation ⇒ voltage change in transformer with loading.

→ The voltage variation in a transformer on load depends on load power factor, load current, total resistance and total leakage reactance of the transformer.

Voltage Regulation:-

$$\boxed{\text{Percentage Regulation} = \frac{V_{2(\text{noload})} - V_{2(\text{full load})}}{V_{2(\text{no-load})}}}$$

$$\% \text{ Regulation} = \frac{\text{Total Voltage drop}}{V_2} \times 100\,\%.$$

For calculating regulation,

$$\boxed{\%\ \text{Regulation} = \frac{I_2 R_{02}\cos\phi \pm I_2 X_{02}\sin\phi}{V_2} \times 100}$$

(or)

$$\boxed{\%\ \text{Regulation} = \frac{I_1 R_{01}\cos\phi \pm I_1 X_{01}\sin\phi}{V_1} \times 100}$$

[+ sign for lagging Power Factor
− sign for leading Power Factor]

A 100 KVA, 6.6 KV/415 V single-phase transformer has an effective impedance of $(3+j8)\Omega$ referred to HV side. Estimate the full load voltage regulation at 0.8 PF lagging and 0.8 PF leading.

Solution:-

$V_1 \Rightarrow$ 6.6 KV / 415 $\Leftarrow V_2$

HV side (Primary)

Then

$3+j8 \Rightarrow Z_{01}$

R_{01} X_{01}

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8)$

$= 36.86$

$\boxed{\sin\phi = 0.6}$

$$\boxed{\%\ \text{Regulation} = \frac{I_1 R_{01}\cos\phi \pm I_1 X_{01}\sin\phi}{V_1}}$$

given, $100\ KVA = V_1 I_1 = V_2 I_2$.

$100 \times 10^3 = V_1 I_1$

$I_1 = \dfrac{100 \times 10^3}{6.6 \times 10^3} \Rightarrow 15.15\ A$

0.8 PF lagging :-

$$\% \text{ Regulation} = \frac{15.15 \times 3 \times 0.8 + 15.15 \times 8 \times 0.6}{6.6 \times 10^3} \times 100$$

$$= 1.652\%$$

0.8 PF leading :-

$$\% \text{ Regulation} = \frac{15.15 \times 3 \times 0.8 - 15.15 \times 8 \times 0.6}{6.6 \times 10^3} \times 100$$

$$= -0.5509\%$$

LOSSES IN A TRANSFORMER

There are mainly two kinds of losses in a transformer

(i) core loss (or) Ironloss

(ii) Copper loss (or) I^2R loss (or) ohmic loss.

(i) core loss :- (P_i) [Remains Practically Constant]

→ The core loss occurs in the transformer iron

→ core loss consists of two components 1. Hysteresis loss.
2. Eddy current loss.

$$\boxed{P_i = P_h + P_e}$$

Hysteresis loss :- (P_h)

→ The alternating flux gets set up in the core and it undergoes cycle of magnetisation and demagnetisation. Therefore, loss of energy occurs in this process due to hysteresis. This loss is called hysteresis loss.

Expression :

$$\boxed{P_h = K_h\, B_{max}^{x}\, f}$$

K_h → Hysteresis constant which depends on the volume and quality of core material.

B_{max} → Maximum flux density in the core

f → frequency of alternating flux.

x → called steinmetz's constant (Varies from 1.5 to 2.5) depending upon the magnetic properties of the core material.
(For iron = 1.6)

→ Hysteresis loss can be reduced by using material that has less area of the hysteresis loop. High grade material like silicon steel has less hysteresis loss.

Eddy Current loss :- (P_e)

This is due to eddy currents in the core and hence Eddy current loss occurs. This loss is called eddy current loss.

Expression, $\boxed{P_e = K_e f^2 B_m^2}$

$K_e \rightarrow$ Eddy current constant, whose value depends on the volume, resistivity of the core material, thickness of laminations.

→ Laminated core is used for reducing eddy current loss.

Copper loss (P_{cu})

The loss of power (wastage of power) in the form of I^2R due to the resistances of the primary and secondary winding is known as copper loss.

Copper loss depends on current flowing through the winding ⇒ depends on load. ⇒ Variable loss.

$$\boxed{P_{cu} \propto I^2 \propto (KVA)^2}$$

If, At full load ⇒ copper loss = I^2R

Then, At half load ⇒ copper loss = $\left(\frac{1}{4}\right) I^2R$

Copper loss is minimum by designing the windings with material having low resistance.

The following losses also present in the transformer in addition to above losses.

Stray load loss :- Leakage field present in the transformer induce eddy currents in tanks, bolts etc...

Dielectric loss :-

loss occurs in the insulating materials.

SEPARATION OF HYSTERESIS AND EDDY CURRENT LOSSES.

The transformer core loss is,

$$P_c = P_h + P_e$$

$$P_c = K_h f B_m^x + K_e f^2 B_m^2 \Rightarrow P_c = A_1 f + A_2 f^2$$

$\div$ by f on both sides,

$$\frac{P_c}{f} = K_h B_m^x + K_e f B_m^2 \Leftarrow \frac{P_e}{f} \Rightarrow \text{proportional to frequency}$$

$\frac{P_h}{f} \Rightarrow$ independent of frequency

Hence the graph.

EMF equation of transformer,

$$V = 4.44 f \phi_m N$$

$$\Rightarrow \frac{V}{f} = 4.44 \phi_m N$$

$$\Rightarrow \frac{V}{f} = 4.44 B_m A_i N \qquad \left[\because B_m = \frac{\phi_m}{A_i}\right]$$

For any transformer, N and Ai are constant.

$$\Rightarrow \frac{V}{f} \propto B_m$$

For a constant $\frac{V}{f}$ (or) B_m value :- $\boxed{\frac{P_c}{f} = A_1 + A_2 f}$

where,
$$A_1 = K_h B_m^x$$
$$A_2 = K_e B_m^2$$

→ The values of A_1 and A_2 can be determined by open circuit test on the transformer

→ During this test $\frac{V}{f}$ and therefore B_m is maintained constant.

→ An adjustment of the speed of the alternator $\left(f = \frac{PN}{120}\right)$ and its excitation help in Keeping ratio $\left(\frac{V}{f}\right)$ Constant.

→ Wattmeter reading gives the iron loss of the transformer. (P_i)

→ P_i, V and f are recorded.

→ $\frac{P_c}{f}$ is plotted against frequency.

→ The intercept of the straight line on the vertical axis gives the constant A_1

→ The slope of the line AB gives the constant A_2.

→ once the constants (A_1 & A_2) are known, hysteresis and eddy current losses can be determined seperately.

An open circuit test of a transformer gave the following data.

Phase voltage in V	214	171	128.4	85.6
Frequency in Hz	50	40	30	20
Power input in W	100	72.5	50	30

Determine the hysteresis and eddy current losses (a) 60Hz (b) 40Hz

Solution:-

$\frac{V}{f}$ is constant = 4.28.

Therefore eqn $\boxed{\frac{P_c}{f} = A_1 + A_2 f}$ can be used.

f	50	40	30	20
$\frac{P_i}{c}$	0.667	0.605	0.556	0.5

Plot f vs $\frac{P_i}{c}$ in graph.

From graph

$A_1 = 0.39$

$A_2 = 0.00554$.

(a) 60Hz

$P_h = (0.39)(60)$

$= 23.4\text{ W}$

$P_e = (0.00554)(60)^2$

$= 19.95\text{ W}$

Total loss at 60Hz $= 23.4 + 19.95$

$= 43.35\text{ W}$

(b) 40Hz

$P_h = (0.39)(40)$

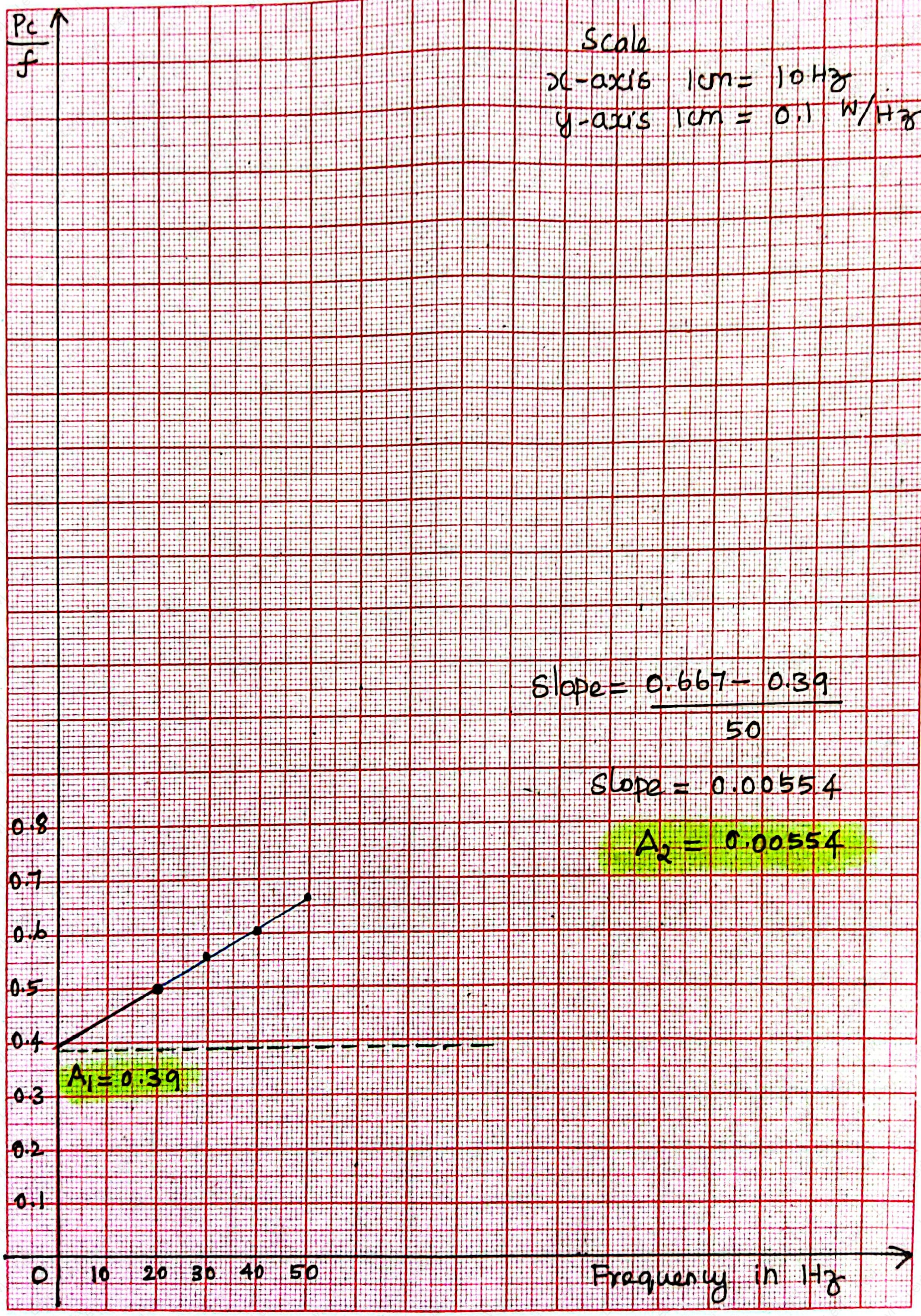
$= 15.6\text{ W}$

$P_e = (0.00554)(40)^2$

$= 8.86\text{ W}$

Total loss at 40Hz $= 15.6 + 8.86$

$= 24.46\text{ W}$



Scale
x-axis 1cm = 10 Hz
y-axis 1cm = 0.1 W/Hz

$$\text{Slope} = \frac{0.667 - 0.39}{50}$$

Slope = 0.00554

$A_2 = 0.00554$

The total core loss of a specimen of silicon steel is found to be 1500W at 50Hz. Keeping the flux constant the loss becomes 3000W. when the frequency is raised to 75 Hz. Calculate seperately the hysteresis and eddy current loss at each of these frequencies.

Solution:-

given :-

$P_{i1} = 1500\,W$; $f_1 = 50Hz$;

$P_{i2} = 3000\,W$; $f_2 = 75Hz$

$$P_i = P_h + P_e$$

$$P_h = K_h B_{max}^{1.6} f = A_1 f$$

$$P_e = K_e B_{max}^{2} f^2 = A_2 f^2$$

50^2

$$\therefore P_{i1} = A_1 f_1 + A_2 f_1^2 \Rightarrow 1500 = 50 A_1 + 2500\, A_2 \quad \text{—①}$$

$$P_{i2} = A_1 f_2 + A_2 f_2^2 \Rightarrow 3000 = 75 A_1 + 5625\, A_2 \quad \text{—②}$$

75^2

solving ① & ②

$$A_1 = 10 \qquad A_2 = 0.4$$

Hence,

50Hz

$P_h = 10 \times 50 \Rightarrow 500\,W$

$P_e = 0.4 \times 50^2 \Rightarrow 1000\,W$

75Hz

$P_h = 10 \times 75 \Rightarrow 750\,W$

$P_e = 0.4 \times 75^2 \Rightarrow 2250\,W$

The coreloss (hysteresis + eddy current loss) for a given specimen of magnetic material is found to be 2000 W at 50Hz. Keeping the flux density constant, the frequency of the supply is raised to 75Hz resulting in a core loss of 3200W. Compute separately hysteresis and eddy current losses at both the frequencies.

Solution:-

given :-

At 50 Hz $\quad P_{i1} = 2000\ W$

At 75 Hz $\quad P_{i2} = 3200\ W$

$$P_h = K_h B_{max}^{1.6} f \Rightarrow P_h \propto f$$

$$P_h = A_1 f$$

$$P_e = K_e B_{max}^2 f^2 \Rightarrow P_e \propto f^2$$

$$P_e = A_2 f^2$$

$$P_{i1} = A_1 f_1 + A_2 f_1^2 \Rightarrow 2000 = 50 A_1 + 2500 A_2 \quad \text{—— (1)}$$

$$P_{i2} = A_1 f_2 + A_2 f_2^2 \Rightarrow 3200 = 75 A_1 + 5625 A_2 \quad \text{—— (2)}$$

solving (1) & (2)

$$A_1 = 34.667 \qquad A_2 = 0.10667$$

Hence,

50Hz

$$P_h = 34.667 \times 50 \Rightarrow 1733.3\ W$$

$$P_e = 0.10667 \times 50^2 \Rightarrow 266.67 W$$

75 Hz

$$P_h = 34.667 \times 75 \Rightarrow 2600\ W$$

$$P_e = 0.10667 \times 75^2 \Rightarrow 600\ W$$

When a transformer is supplied at 400V, 50Hz the hysteresis loss is found to be 310W and eddy current loss is found to be 260W. Determine the hysteresis loss and eddy current loss when the transformer is supplied at 800V, 100Hz.

solution:-

given,

$P_h = 310W$ } 400V and 50Hz
$P_e = 260W$

To Find

P_h & P_e when supplied at 800V & 100Hz

Check :- flux density ⇒ constant

I st case

$\frac{V}{f} = \frac{400}{50}$

$= 8$ //

II nd case

$\frac{V}{f} = \frac{800}{100}$

$= 8$ /

∴ flux density is constant.

Hence A_1 and A_2 are constant in both the case.

Find A_1 & A_2

$\boxed{P_h = A_1 f}$

$\boxed{P_e = A_2 f^2}$

From given data, $310 = A_1(50)$

$A_1 = 6.2$ //

$260 = A_2(50)^2$

$A_2 = 0.104$ //

Now,

At 100Hz

$P_h = 6.2(100)$

$= 620W$ //

$P_e = 0.104(100)^2$

$= 1040W$ //

OPEN CIRCUIT AND SHORT CIRCUIT TESTS

Purpose of this test,

→ Predetermination of Efficiency and Regulation

↳ The efficiency and regulation of a transformer on anyload condition and at any powerfactor can be predetermined. (the actual load is not used.)

→ Equivalent circuit parameters of a transformer.

↳ Opencircuit test (o.c. test)

✓ used to determine (Iron loss (or) core loss (or) Noload loss (or) constant loss) (P_i)

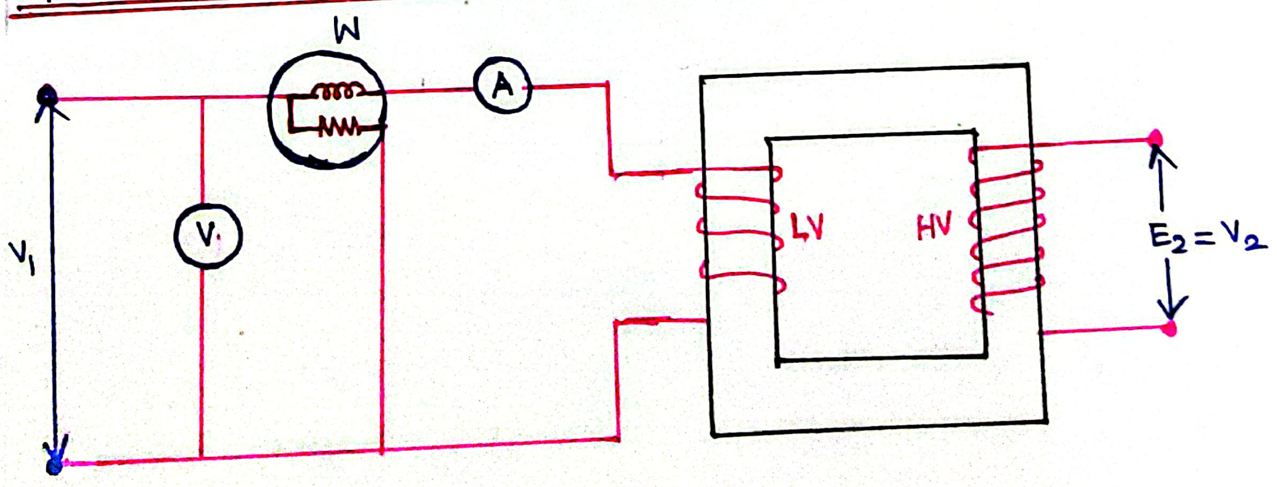
✓ we get the values for R_o, X_o, I_o – I_w, I_μ (referred to meter side)

↳ Short circuit test (S.C. test)

✓ Full-load copper loss (P_{cu})

✓ Equivalent resistance and reactance (Referred to meter side)

Open circuit test (or) No-Load test :-



Based on convenient,

→ Any one of the winding of the transformer is left open, but usually high voltage is left open - circuited

→ The rated voltage is given to L.V winding (Voltmeter reading = rated voltage) Then, all the three instrument readings are recorded.

Ammeter reading (I_0) ⇒ No load current

Voltmeter reading (V_0) ⇒ Input voltage (rated)

Wattmeter reading (P_0)

Core loss (same for all loads) + ohmic loss ($I^2 R_1$)

⇓

✓ No load current is very small 2 to 6% of rated current

✓ ∴ $I^2 R$ is negligibly small in Primary and Nil in secondary winding (no current)

✓ Hence the Wattmeter reading (P_0) can be taken equal to transformer core-loss.

Calculations:-

$$P_0 = V_0 I_0 \cos\phi_0$$

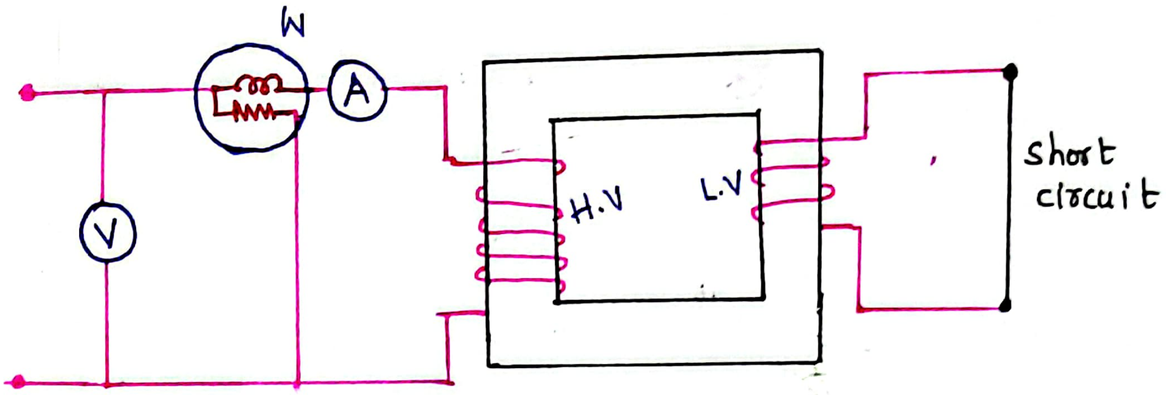
$$\cos\phi_0 = \frac{P_0}{V_0 I_0}$$

$$I_\mu = I_0 \sin\phi_0 \quad , \quad I_W = I_0 \cos\phi_0$$

$$X_0 = \frac{V_1}{I_\mu} \qquad R_0 = \frac{V_1}{I_W}$$

$$I_0 = \sqrt{I_W^2 + I_\mu^2}$$

Short circuit test :-



→ usually Low voltage side of the transformer is short circuited.

→ The applied voltage is adjusted by auto-transformer, to circulate rated current in H.V side
(A primary voltage of 2 to 12% of its rated value is sufficient to circulate rated currents in both primary and secondary windings)

→ Since applied voltage is very low, flux linking with the core is very small. So Iron loss is so small and can be neglected. Thus the wattmeter reading, in short circuit test, gives the full load copper loss.

→ Instrument readings are recorded as. V_{sc}, I_{sc}, P_{sc}

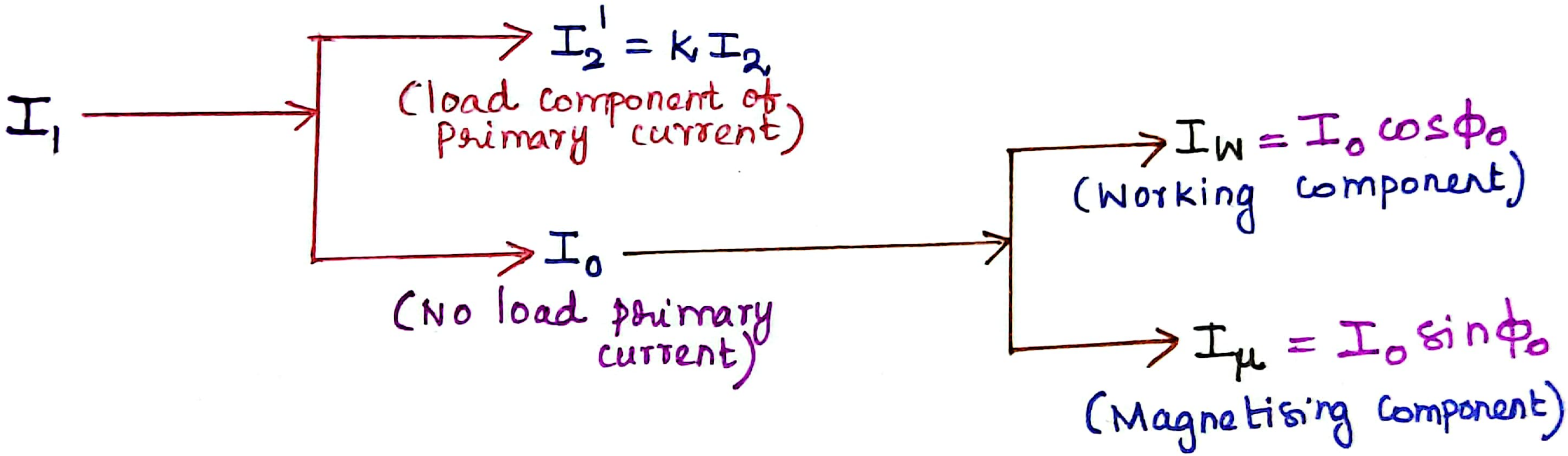
→ Calculation :-

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$P_{sc} = I_{sc}^2 R_{01}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

The components of primary current (I_1) are.



→ When secondary parameters are referred to primary,

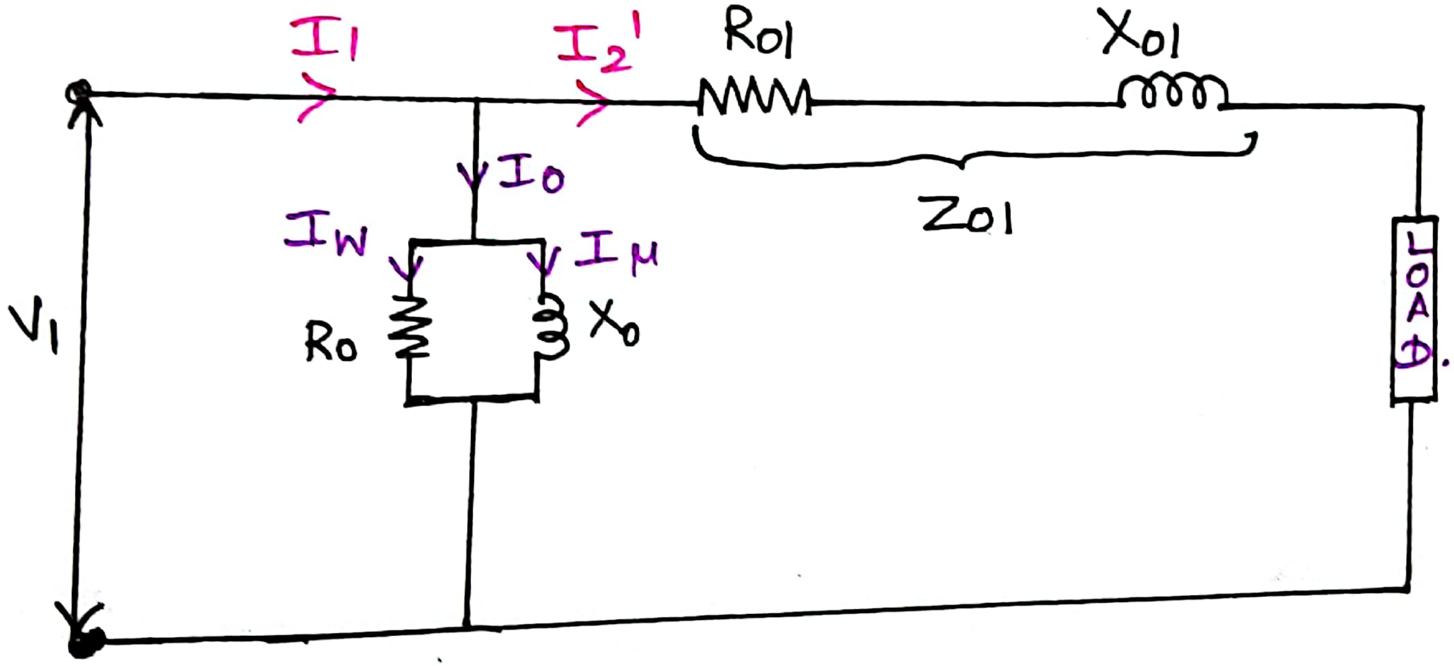
- ✓ Resistances and Reactances are divided by K^2
- ✓ Voltages are divided by K
- ✓ Currents are multiplied by K

$$R_2' = \frac{R_2}{K^2} \quad ; \quad X_2' = \frac{X_2}{K^2} \quad ; \quad Z_L' = \frac{Z_L}{K^2}$$

$$I_2' = K I_2 \quad ; \quad V_2' = \frac{V_2}{K}$$

$R_1 + R_2' \Rightarrow R_{01} \Rightarrow$ Equivalent resistance referred to Primary.

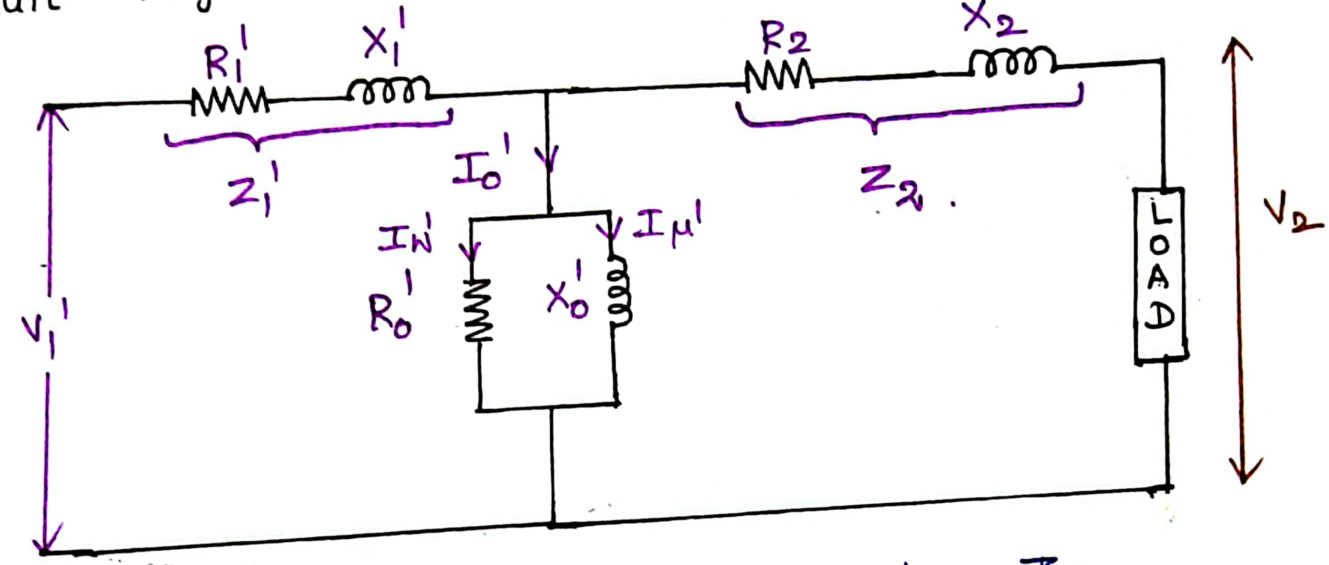
$X_1 + X_2' \Rightarrow X_{01} \Rightarrow$ Equivalent reactance referred to primary.



$$R_0 = \frac{V_1}{I_W}$$

$$X_0 = \frac{V_1}{I_\mu}$$

All the primary value can be referred to secondary and we can obtain the equivalent circuit referred to secondary.



$R_1' = K^2 R_1$

$X_1' = K^2 X_1$

$Z_1' = K^2 Z_1$

$V_1' = K V_1$

$I_0' = \frac{I_0}{K}$

$I_W' = \frac{I_W}{K}$

$I_\mu' = \frac{I_\mu}{K}$

Obtain the approximate equivalent circuit of a given 200/2000 V single-phase 30KVA transformer having the following test results :

O.C test : 200V, 6.2A, 360 W on L.V side.

S.C test : 75V, 18 A, 600 W on H.V side.

Solution:-

O.C test (L.V side) : (Primary)
meter

$V_o = 200V$

$I_o = 6.2 A$

$P_o = 360 W$

$$\boxed{P_o = V_o I_o \cos\phi_o}$$

$$\cos\phi_o = \frac{P_o}{V_o I_o}$$

$$= \frac{360}{200 \times 6.2}$$

$\cos\phi_o = 0.290$ //

$\phi_o = \cos^{-1}(0.290) \Rightarrow \phi_o = 73.12°$

$\sin\phi_o = 0.957$ //

Working Component, $\boxed{I_W = I_o \cos\phi_o}$

$I_W = 6.2 \times 0.290$

$$\boxed{I_W = 1.798 \text{ A}}$$

Magnetizing Component, $\boxed{I_\mu = I_o \sin\phi_o}$

$I_\mu = 6.2 \times 0.957$

$$\boxed{I_\mu = 5.93 \text{ A}}$$

200/2000 (given)

V_1 V_2

$$\boxed{R_0 = \frac{V_1}{I_W}}$$

$$= \frac{200}{1.798}$$

$$\boxed{R_0 = 111.23\ \Omega}$$

$$\boxed{X_0 = \frac{V_1}{I_\mu}}$$

$$= \frac{200}{5.93}$$

$$\boxed{X_0 = 33.72\ \Omega}$$

S.C test (H.V. side) meter : (secondary side)

$V_{sc} = 75\,V$

$I_{sc} = 18\,A$

$P_{sc} = 600\,W$

$$\boxed{Z_{02} = \frac{V_{sc}}{I_{sc}}}$$

$$= \frac{75}{18}$$

$$\boxed{Z_{02} = 4.167\ \Omega}$$

$$\boxed{P_{sc} = I_{sc}^2 R_{02}}$$

$$R_{02} = \frac{P_{sc}}{I_{sc}^2}$$

$$= \frac{600}{18^2}$$

$$\boxed{R_{02} = 1.85\ \Omega}$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$$

$$= \sqrt{4.167^2 - 1.85^2}$$

$$\boxed{X_{02} = 3.733\ \Omega}$$

Equivalent circuit referred to L.V side (Primary) :-

$$\boxed{Z_{01} = \frac{Z_{02}}{K^2}}$$

$$= \frac{4.167}{10^2}$$

$$Z_{01} = 0.04167\ \Omega$$

$$\boxed{K = \frac{V_2}{V_1}}$$

$$= \frac{2000}{20}$$

$$\boxed{K = 10}$$

$$\boxed{R_{01} = \frac{R_{02}}{K^2}}$$

$$= \frac{1.85}{10^2}$$

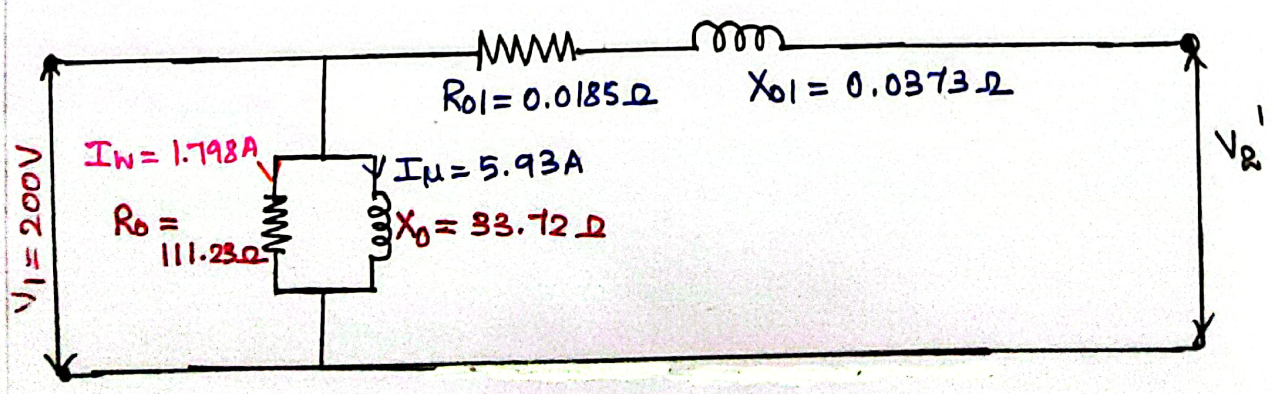
$$R_{01} = 0.0185\ \Omega.$$

$$\boxed{X_{01} = \frac{X_{02}}{K^2}}$$

$$= \frac{3.733}{10^2}$$

$$X_{01} = 0.0373\ \Omega$$

Approximate equivalent circuit.



The following data were obtained on a 20kVA, 50Hz 2000/200 V distribution transformer:-

	Voltage (V)	current (A)	Power (W)
OC test with HV open-circuited	200	4	120
SC test with LV short-circuited	60	10	300

Draw the approximate equivalent circuit of the transformer referred to the (i) HV side (ii) LV side.

Solution:-

O.C test (L.V. side meter):- (secondary)

$V_0 = 200$ V

$I_0 = 4$ A

$P_0 = 120$ W

$$\boxed{P_0 = V_0 I_0 \cos\phi_0}$$

$$\cos\phi_0 = \frac{P_0}{V_0 I_0}$$

$$= \frac{120}{200 \times 4}$$

$$\cos\phi_0 = 0.15 \quad \text{(lag)}$$

$\phi_0 = \cos^{-1}(0.15)$

$\phi_0 = 81.37$

$\sin\phi_0 = 0.988$

$$\boxed{I_W' = I_0 \cos\phi_0}$$

$$= 4 \times 0.15$$

$$I_W' = 0.6 \text{ A}$$

$$I_W' = \frac{I_W}{K}$$

$$I_W = K I_W'$$

$$\boxed{I_\mu' = I_0 \sin\phi_0}$$

$$= 4 \times 0.988$$

$$I_\mu' = 3.94 \text{ A}$$

$$I_\mu' = \frac{I_\mu}{K}$$

$$I_\mu = K I_\mu'$$

$$\boxed{K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1}$$

$I_W = 0.1 \times 0.6$

$I_W = 0.06\ A$

$I_\mu = 0.1 \times 3.94$

$I_\mu = 0.394\ A$

S.C test (H.V side meter) (Primary)

$V_{sc} = 60\ V$

$I_{sc} = 10\ A$

$P_{sc} = 300\ W$

$$\boxed{Z_{01} = \frac{V_{sc}}{I_{sc}}}$$

$$= \frac{60}{10}$$

$$Z_{01} = 6\,\Omega$$

$$\boxed{P_{sc} = I_{sc}^2 R_{01}}$$

$$R_{01} = \frac{P_{sc}}{I_{sc}^2}$$

$$= \frac{300}{10^2}$$

$$R_{01} = 3\,\Omega$$

$$\boxed{X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}}$$

$$= \sqrt{6^2 - 3^2}$$

$$X_{01} = 5.196\,\Omega$$

$$\boxed{R_0 = \frac{V_1}{I_W}}$$

$\underset{V_1}{2000} / \underset{V_2}{200}$

$$R_0 = \frac{200}{0.06} = 33.3\ k\Omega$$

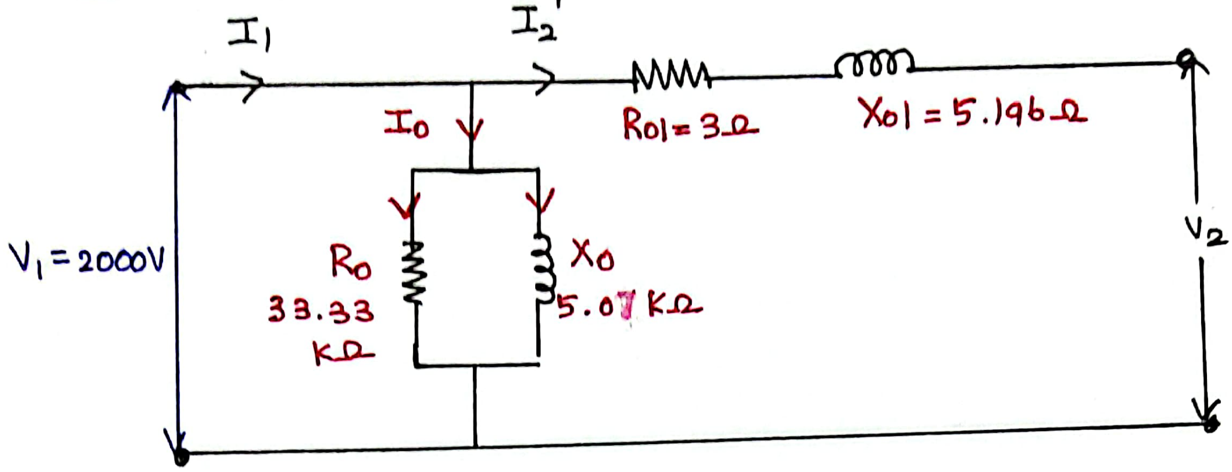
$$\boxed{X_0 = \frac{V_1}{I_\mu}}$$

$$X_0 = \frac{2000}{0.394}$$

$$X_0 = 5076.1\,\Omega$$

$$X_0 = 5.07\,k\Omega$$

(i) Equivalent circuit referred to primary (HV side)



(ii) Equivalent circuit referred to secondary (L.V side)

$$\boxed{R_{02} = K^2 R_{01}}$$

$$= 0.1^2 \times 3$$

$$R_{02} = 0.03\,\Omega$$

$$\boxed{X_{02} = K^2 X_{01}}$$

$$= 0.1^2 \times 5.196$$

$$X_{02} = 0.5196\,\Omega$$

$$\boxed{R_0' = K^2 R_0}$$

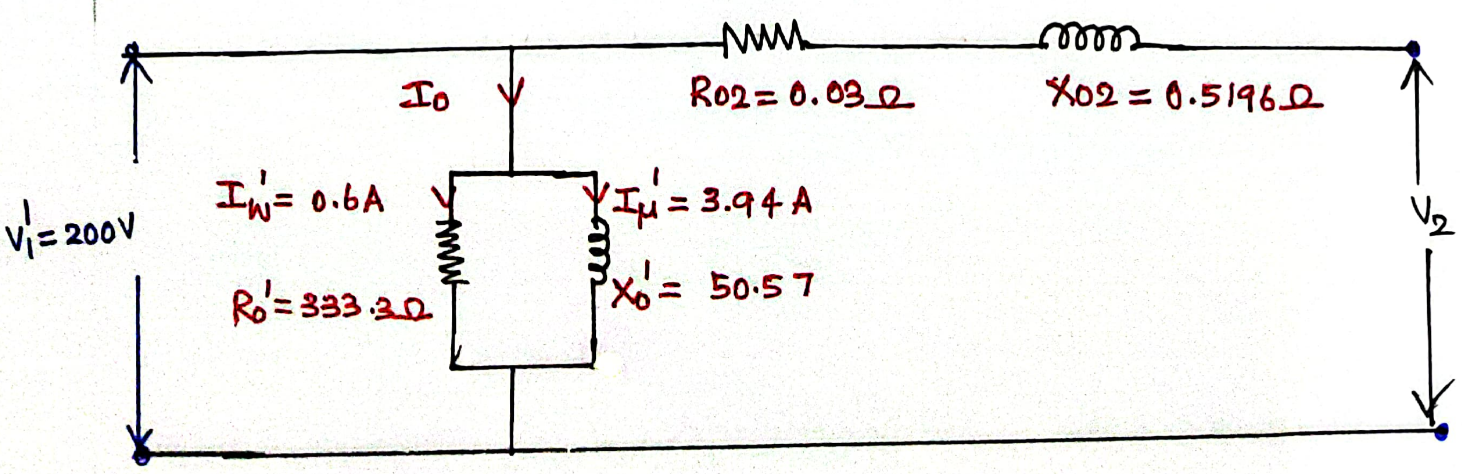
$$= 0.1^2 \times 33.33 \times 10^3$$

$$R_0' = 333.3\,\Omega$$

$$\boxed{X_0' = K^2 X_0}$$

$$= 0.1^2 \times 5.07 \times 10^3$$

$$X_0' = 50.57\,\Omega.$$



Consider a 20KVA, 2200/220 V, 50Hz transformer. The oc/sc test results are as follows.

O.C Test : 220V, 4.2A, 148 W (L.V side)

S.C Test : 86V, 10.5 A, 360 W (H.V side).

(i) Determine the regulation at 0.8 PF lagging at full load.

(ii) What is the P.F on short circuit.

Solution:-

Short circuit test: (HV side meter)

$V_{sc} = 86\,V$, $I_{sc} = 10.5\,A$, $P_{sc} = 360\,W$

short circuit test has been conducted on H.V (primary) side.

$$\boxed{\%\ \text{Regulation (lag)} = \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1}}$$

$$\boxed{Z_{01} = \frac{V_{sc}}{I_{sc}}}$$

$$= \frac{86}{10.5}$$

$$Z_{01} = 8.19\,\Omega$$

$$P_{sc} = I_{sc}^2 R_{01}$$

$$R_{01} = \frac{P_{sc}}{I_{sc}^2}$$

$$= \frac{360}{(10.5)^2}$$

$$R_{01} = 3.265\,\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

$$= \sqrt{8.19^2 - 3.265^2}$$

$$X_{01} = 7.51\,\Omega$$

given, 20 KVA

$$20\,KVA = V_1 I_1 = V_2 I_2$$

$$20 \times 10^3 = V_1 I_1$$

$$I_1 = \frac{20 \times 10^3}{2200} \qquad \left[\because \overset{V_1}{2200} / \overset{V_2}{220}\right]$$

$$I_1 = 9.09\ A$$

(i)

Regulation at 0.8 P.F

$$\%\ Regulation = \frac{9.09\,(3.27 \times 0.8 + 7.51 \times 0.6)}{2200} \times 100\,\%$$

$$\boxed{\%\ Regulation = 2.94\,\%}$$

(ii) P.F on short circuit.

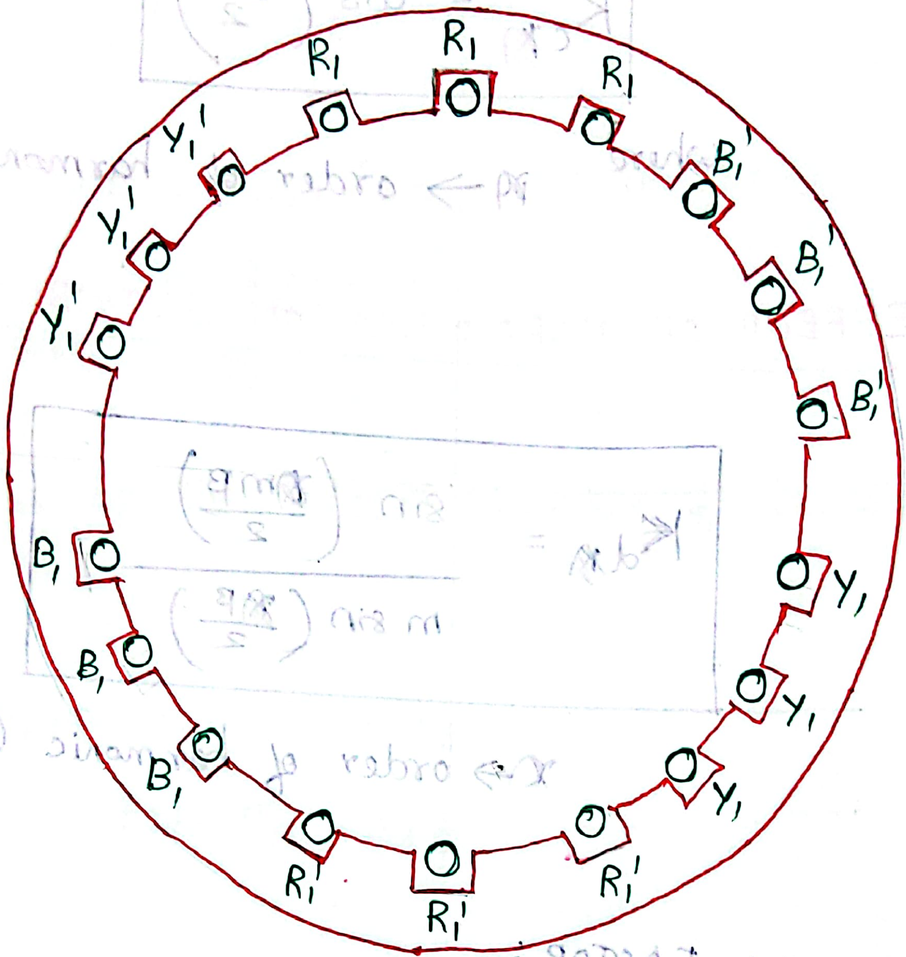
$$P_{sc} = V_{sc} I_{sc} \cos\phi_{sc}$$

$$\cos\phi_{sc} = \frac{P_{sc}}{V_{sc} I_{sc}}$$

$$= \frac{360}{86 \times 10.5}$$

$$\boxed{\cos\phi_{sc} = 0.398}$$

MMF OF DISTRIBUTED WINDINGS.

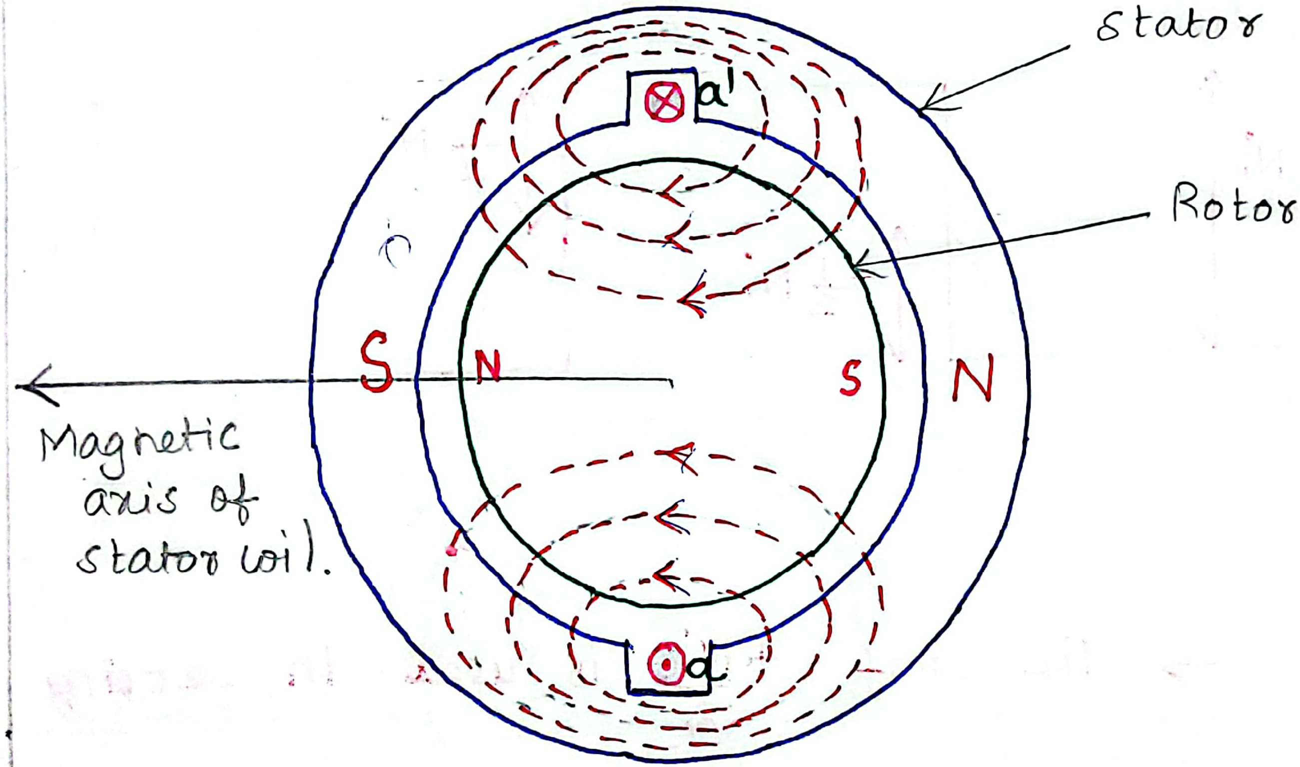


1. cylinderical rotor machine.
2. Permeability of rotor and stator is much greater than that of air (made by using high grade material)
3. The magnetic flux lines are assumed to cross the airgap radially.

MMF OF A SINGLE COIL :

Consider a cylinderical rotor machine with small airgap as shown in Fig.

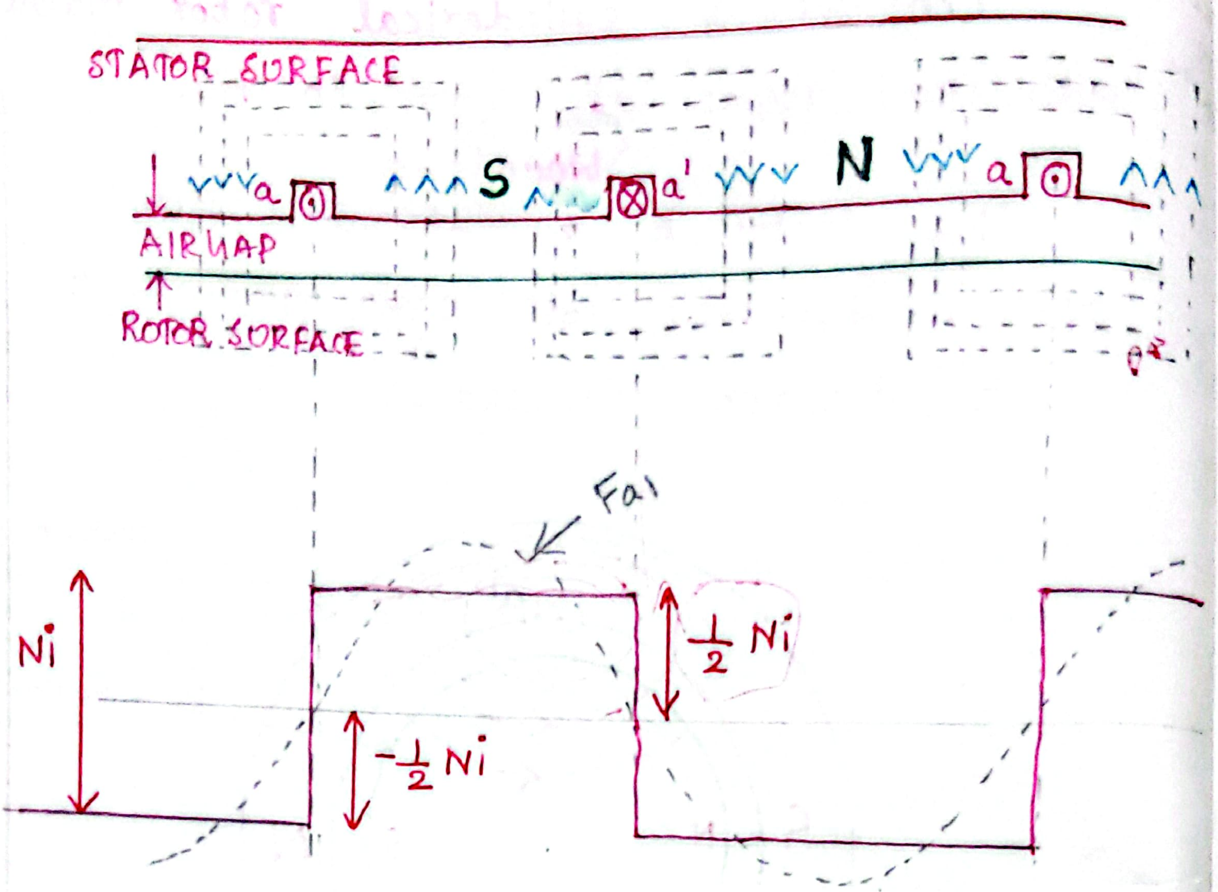
The stator is wound for 2poles. With a N-turn, full pitch coil carrying current 'i' in the direction indicated.



This current sets up magnetic flux. (shown by dotted lines).

The rotor must have opposite poles. to maintain synchronism.

With 'N' number of turns and 'i' stator current, the MMF = Ni prevails in the space.



→ The mmf $+\frac{Ni}{2}$ is used in setting flux from rotor to stator.

→ The m.mf $-\frac{Ni}{2}$ is used in setting flux from stator to rotor.

The rectangular m.mf wave $\frac{1}{2}$Ni can be resolved by its Fourier series into fundamental and higher order harmonic components.

The fundamental Component of M.M.F wave is given by :

$$\boxed{F_{a1} = \frac{4}{\pi} \frac{Ni}{2} \cos\theta}$$

$$\boxed{F_{a1(peak)} = \frac{4}{\pi} \frac{Ni}{2}}$$